INTERPOLATION SERIES IN LOCAL FIELDS OF PRIME CHARACTERISTIC

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1. Introduction. In 1944 Dieudonné [3] proved a p-adic analogue of the Weierstrass Approximation Theorem by an inductive argument involving the polynomial approximation of certain continuous characteristic functions. In 1958 Mahler [4] proved the sharper result that each continuous p-adic function f defined on the p-adic integers is the uniform limit of the "interpolation series"

$$f(t) = \sum_{n=0}^{\infty} \Delta^n f(0) \binom{t}{n},$$

where

$$\Delta^n f(0) = \sum_{k=0}^{n} (-1)^k \binom{n}{k} f(n-k).$$

The crucial step in Mahler's proof involves showing that $\lim_{n\to\infty} \Delta^n f(0) = 0$ for the p-adic topology, and he demonstrates this by passing to a certain cyclotomic extension of the rationals. In fact, this follows quickly from Dieudonné's theorem for if $p(t)$ is a polynomial of degree r for which $|f(t) - p(t)|_p < \epsilon$ for $t \in Z_p$, then $|\Delta^n f(0) - \Delta^n p(0)|_p < \epsilon$ for all n. Hence if $n > r$, $\Delta^n p(0) = 0$ and $|\Delta^n f(0)|_p < \epsilon$.

In the present paper we use the above idea to simplify our earlier proof of a Mahler type theorem for continuous functions on the ring V of formal power series over a finite field $GF(q)$ [5]. Although the proof by Dieudonné admits a straightforward generalization to any locally compact non-archimedean field, in this case we accomplish the polynomial approximation of the relevant characteristic functions without recourse to induction by using certain powers of the Carlitz polynomials $G'_{q^r-1}(t)/g_{q^r-1}$ [1]. We conclude by giving a sufficient condition for the differentiability of a function f defined on V.

2. Preliminaries. Let $GF\,[q, x]$ be the ring of polynomials over the finite field $GF(q)$ of characteristic p and let $GF(q, x)$ be the quotient field of $GF[q, x]$. Denote by V the ring of formal power series over $GF(q)$ and by F the field of formal power series over $GF(q)$. Set $|0| = 0$. If $\alpha \in F - \{0\}$ is given by

$$\alpha = \sum_{i=-\infty}^{\infty} a_i x^i, \tag{2.1}$$

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