SOME EXTENSIONS OF THE MEHLER FORMULA, II

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1. In a recent paper [5] we gave the formula

$$(1)\qquad \sum_{m,n,p=0}^{\infty} H_{n+p+r}(x)H_{m+p+s}(y)H_{m+n}(z)\frac{u^m}{m!}\frac{v^n}{n!}\frac{w^p}{p!}$$
$$= S\sum_{k=0}^{\min(r,s)} 2^{2k}k!\binom{r}{k}\binom{s}{k}\left(\frac{w-2uv}{\sqrt{\{(1-4u^2)(1-4v^2)\}}}\right)^k$$
$$\cdot H_{r-k}\left(\frac{(x-2vz)(1-4u^2)-2(y-2uz)(w-2uv)}{\sqrt{\{\Delta(1-4u^2)\}}}\right)$$
$$\cdot H_{s-k}\left(\frac{(y-2uz)(1-4v^2)-2(x-2vz)(w-2uv)}{\sqrt{\{\Delta(1-4v^2)\}}}\right),$$

where, for convenience,

$$(2)\qquad \Delta = 1-4u^2-4v^2-4w^2+16uvw,$$

$$(3)\qquad S = \Delta^{-\frac{1}{2}(r+s+1)}(1-4u^2)^{r/2}(1-4v^2)^{s/2}$$
$$\cdot\exp\left\{\sum x^2-\frac{1}{\Delta}\left(\sum x^2-4\sum u^2x^2-4\sum wxy+8\sum uvxy\right)\right\}$$

and where $\sum x^2$, $\sum u^2x^2$, $\sum wxy$, $\sum uvxy$ are symmetric functions in the indicated variables and $H_n(z)$ denotes the classical Hermite polynomial defined by Rodrigues' formula

$$(4)\qquad H_n(z) = (-1)^n \exp(x^2)\, D_x^n \exp(-x^2),\quad D_x = d/dx.$$

Formula (1) provides an elegant unification of several extensions of the well-known Mehler formula (cf., e.g., [3; 198]) given recently by Carlitz [1] and [2].

The present note is a sequel to our paper [5]. We first derive the formula

$$(5)\qquad \sum_{m,n,p=0}^{\infty} H_{m+p}(x)H_{n+p}(y)H_{m+r}(z)H_{n+s}(t)\frac{u^m}{m!}\frac{v^n}{n!}\frac{w^p}{p!}$$
$$= R(1-4v^2-4w^2)^{r/2}(1-4u^2-4w^2)^{s/2}$$
$$\cdot\sum_{k=0}^{\min(r,s)} k!\binom{r}{k}\binom{s}{k}\left(\frac{16uvw}{\sqrt{\{(1-4u^2-4w^2)(1-4v^2-4w^2)\}}}\right)^k$$

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