ON σ-TYPE POLYNOMIALS GENERATED BY $A(t)\psi(xH(t))$

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Huff and Rainville [3] proved that if the polynomial set $\{p_n(x)\}$ is generated by $A(t)\psi(xt)$, then a necessary and sufficient condition that $\{p_n(x)\}$ be a Sheffer A-type m, $m > 0$, is

$$\psi(xt) = {}_0F_q[—; (b_q); \alpha xt],$$

where α is a nonzero constant. Goldberg [2] generalising the above result proved that if the polynomial set $\{p_n(x)\}$ is generated by $A(t)\psi(xH(t))$, then a necessary and sufficient condition for $\{p_n(x)\}$ to be a Sheffer A-type m, $m > 0$, is that there exist a positive number r which divides m and numbers $b_1, b_2, \cdots, b_r$ (none zero nor negative integers) such that $\{p_n(x)\}$ is σ-type zero for

$$\sigma = D \prod_{k=1}^{r} (xD + b_k - 1), \qquad D \equiv \frac{d}{dx},$$

and the inverse $H^{-1}(t)$ of $H(t)$, i.e., $H(H^{-1}(t)) = H^{-1}(H(t)) = t$, is a polynomial of degree $s = m/r$ exactly. (For notations and the properties of Sheffer A-type and σ-type polynomials see [4].)

In this note we prove the following extension of the above results. A necessary and sufficient condition that the polynomial set $\{p_n(x)\}$ generated by $A(t)\psi(xH(t))$ is σ-type m, $m > 0$, with $\sigma = D \prod_{i=1}^{q} (xD + b_i - 1)$ is that there exist a positive integer r which divides m and numbers $\gamma_1, \gamma_2, \cdots, \gamma_r$ (which are nonzero and nonnegative integers) such that $\{p_n(x)\}$ is σ^*-type zero for

$$\sigma^* = D \prod_{i=1}^{q} (xD + b_i - 1) \prod_{j=1}^{r} (xD + \gamma_j - 1)$$

and $H^{-1}(t)$ is a polynomial of degree $s = m/r$ exactly. To this end suppose

$$\sum_{n=0}^{\infty} p_n(x)t^n = A(t)\psi(xH(t)), \tag{1}$$

where

$$A(t) = \sum_{n=0}^{\infty} a_n t^n \qquad a_0 \neq 0$$

$$\psi(t) = \sum_{n=0}^{\infty} \psi_n t^n \qquad \psi_n \neq 0, \quad n \geq 0$$

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