LOCALLY GEOMETRICALLY UNKNOTTED ONE-MANIFOLDS

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1. Introduction. It has been known for some years [3] that an arc or simple closed curve in R^3 or S^3 which is both LU and LPU is tame. Miller [4] has introduced the notion of a subarc A of a one-manifold J in S^3 being geometrically knotted relative to J. This concept has a rather natural local form which will herein be denoted as J is locally geometrically unknotted at p (LGU at p). While Miller did not explicitly formulate this local concept, one of his theorems can be stated in terms of it as follows. If a one-manifold J in S^3 is LU at an interior point p, then it is LGU at p.

The main results obtained here are the following.

THEOREM I. *A simple closed curve in S^3 is tame if and only if it is both* LPU *and* LGU.

THEOREM II. *If A is an* LPU *arc in S^3, then the following are equivalent.*
1. *A is tame.*
2. *A is* LU *at each interior point.*
3. *A is* LGU *at each interior point.*

It will also be shown that LPU at p and LGU at p are independent properties and that no arc is both LPU and LGU, since no tame arc can be LGU at an endpoint.

2. Definitions and notation. A space is called a crumpled cube if it is topologically the closure of the bounded component of the complement of a 2-sphere in R^3. If D is a cell, then $\dot{D}$ is the combinatorial boundary of D and $\mathring{D}$ is $D \setminus \dot{D}$.

An arc or simple closed curve J in S^3 or R^3 is said to be locally unknotted at $p \in J$ (LU at p) if some neighborhood of p in J lies on the boundary of a disk. It is said to be locally peripherally unknotted at p (LPU at p) if p has arbitrarily small crumpled cube neighborhoods whose boundary 2-spheres meet J in one or two points according as p is an endpoint or an interior point of J. Further, J is said to be LU (LPU) if it is LU at p (LPU at p) for each p in J [2].

Let C and H be 3-balls in S^3 with Int $H \subset$ Int C and Bdry $H \cap$ Bdry $C = D_1 \cup D_2$, where D_1 and D_2 are a disjoint pair of disks. Then the closure M of $C \setminus H$ is said to be a cube with a hole if $S^3 \setminus$Int M is a tame solid torus. According as the core of this torus is a trivial or nontrivial knot, M is said to have an unknotted or knotted hole.

Following Miller [4] a connected one-manifold J will be said to span a cube with a hole M if $J \cap M = \emptyset$ while $J \cap$ Int H has an odd number of components

Received October 28, 1971. Revisions received December 13, 1971.