TOPOLOGICAL GROUPS WHICH ARE NOT FULL HOMEOMORPHISM GROUPS

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1. Introduction. In [3] J. de Groot proves that for every group H and any positive integer n, there exists a complete, connected, locally connected, metric space X of dimension n such that $G(X)$ is isomorphic to H. The main purpose of this paper is to show that this result does *not* extend to *topological* groups. Our main theorem asserts that if X is metric and admits a flow, then $G(X)$ is infinite dimensional. (It should be noted that J. E. Keesling has improved the results of this paper. He proves in [5] that if X is metric and $G(X)$ is locally compact, then $G(X)$ must be zero dimensional.)

We also construct examples to show that if G is the direct product of finite cyclic groups, then there *does* exist a metric continuum X_G whose group of homeomorphisms is algebraically and topologically the same as G.

I would like to thank Professors A. D. Wallace and J. E. Keesling for some helpful (related and non-related) conversations.

2. The main theorem. The proof of the theorem below is essentially the same as the author's proof [2] that the group of homeomorphisms of a manifold is infinite dimensional. An important tool is the following theorem of A. Beck [1]. A metric space admits a flow with an arbitrary closed set as its fixed point set iff it admits a fixed point free flow.

THEOREM 2.1. *Let X be a metric space which admits a flow. Then $G(X)$ contains a Hilbert cube and is therefore infinite dimensional.*

Proof. Let $T = \{g_t \mid t \in R\}$ be a flow on X. Let F be the fixed point set of T. Then $X - F$ is open in X and admits a fixed point free flow.

Let $\{U_i\}_{i=1}^{\infty}$ be a sequence of open subsets of $X - F$ such that

(1) $\bar{U}_i \cap \bar{U}_j = \varnothing$ for $i \neq j$
(2) $\bar{U}_i \subseteq X - F$ for all i
(3) diam $U_i \to 0$.

By Beck's theorem there exists a flow $T_i = \{g_{i,t} \mid t \in R\}$ on $X - F$ whose fixed point set is $(X - F) - U_i$. Since $\bar{U}_i \subseteq X - F$, this flow can be extended to a flow $S_i = \{h_{i,t} \mid t \in R\}$ on X by defining $h_{i,t}$ to be $g_{i,t}$ on $X - F$ and $h_{i,t}(x) = x$ on F.

For each S_i there exists $s_i \in R$ such that for $0 \leq s < t \leq s_i$, $h_{i,s} \neq h_{i,t}$. Let J_i be the interval $[0, s_i]$ of the flow T_i. Then $H = \prod_{i=1}^{\infty} J_i$ is a Hilbert

Received March 18, 1970. Revised December 13, 1971.