RIEMANNIAN MANIFOLDS WITH NON-NEGATIVE RICCI CURVATURE

BY ANDRÉ AVEZ

1. Introduction. In 1941 S. B. Myers [7] proved that if the Ricci curvature of a complete manifold M is uniformly bounded from below by a positive constant, then M is compact and has finite first homotopy group $\pi_1(M)$. On the other hand Bochner and Lichnerowicz [5] proved that a compact manifold with non-negative Ricci curvature and non-trivial first Betti number $B_1(M) = b$ is locally a Riemannian product $\mathbf{R}^b \times M'$. This suggests the following conjecture. The universal covering $\tilde{M}$ of a compact manifold M with non-negative Ricci curvature and infinite first homotopy group is the Riemannian product of $\mathbf{R}^k$, $k > 0$, by a compact manifold of positive Ricci curvature.

A step toward this conjecture is due to T. Aubin [2]. If the Ricci curvature of a compact manifold is non-negative and positive at some point, then one can find a new metric with strictly positive Ricci curvature. This implies that if the Ricci curvature of a compact manifold M is non-negative and if $\pi_1(M)$ is infinite, then there exists at each point of M a non-vanishing tangent vector u such that $\operatorname{Ric}(u) = 0$.

The present article provides another step. The following statement arises as a consequence of the main results which are stated as Theorems 3.4, 3.7 and 3.11.

THEOREM. *Let M be a compact Riemannian manifold with non-negative Ricci curvature. If for some point $0 \in M$ every geodesic γ: $[0, +\infty[\to M$ issuing from 0 and parametrized by its arc length s has one of the following properties*

(i) $\sup_{s \geq 0} s^{\frac{1}{2}} \cdot \operatorname{Ric}[\dot{\gamma}(s)] = +\infty,$

(ii) $\sup_{s \geq 0} |s^{\frac{3}{2}} \cdot \rho[\dot{\gamma}(s)]| = +\infty,$

then $\pi_1(M)$ is finite; $\operatorname{Ric}[\dot{\gamma}(s)]$ *denotes the Ricci curvature in the direction of the tangent vector $\dot{\gamma}(s) = \gamma^*(d/ds)$ and $\rho[\ ]$ is the sectional curvature in an arbitrary 2-plane containing $\dot{\gamma}(s)$.*

Related results were obtained by W. Ambrose [1] and E. Calabi [4] under integral assumptions, while (i) and (ii) are pointwise conditions.

2. Notations. Throughout this paper we shall use Einstein's convention and Lichnerowicz's notations [5], where R_{ijkl} denotes the components of the curvature tensor; the Ricci tensor has components $R_{ij} = R^k_{ikj}$ (It is positive definite

Received April 11, 1970. This paper was written while the author was supported by contract AEC AT (11-1)-34.