DECOMPOSITIONS OF E^3 INTO SHRINKABLE ENDS

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1. Introduction. An upper semi-continuous decomposition G of a space X is a collection of pairwise disjoint subsets of X whose union is X and such that if g is an element of G and U is an open set in X containing g, then there is an open set V in X with $g \subset V \subset U$ and such that V is a union of elements of G. This paper will deal with upper semi-continuous decompositions whose elements are compact and connected. Such decompositions are called monotone. The set of non-degenerate elements of an upper semi-continuous decomposition G will be denoted by H_G, and E^3/G will denote the decomposition space associated with G.

The following theorem is an important result of this paper which is a direct consequence of Theorem 8 and Corollary 2.

THEOREM 1. *If G is a monotone decomposition of E^3 such that H_G is countable and each element of H_G is a tree consisting of tame arcs, then E^3/G is topologically E^3.*

Recall that a tree is a space homeomorphic to a finite connected one-dimensional simplicial complex containing no simple closed curves. "Consisting of tame arcs" means that each arc of the tree corresponding to a one-simplex is tame. An example given by Fox and Artin [5; 987] shows that this condition on a tree is weaker than requiring the tree to be tame.

Theorem 1 extends a result of Bing [3; 370, Theorem 3] and answers a question posed by Armentrout [1; 5]. A proof of Theorem 1 different from the one given here is outlined in [7]. Although the present proof requires more preliminary work than the one in [7], it is hoped that the methods developed here can be applied elsewhere.

2. Shrinkable ends. In this section we define and give some properties of shrinkable ends. The origin of the term should be apparent from its definition. Throughout this paper the term "map" will mean "continuous function from E^3 onto itself" and Id will denote the identity map.

DEFINITION 1. Let g_0 be an element of an upper semi-continuous decomposition G of E^3 and let C and D be subsets of g_0. C is shrinkable into D if for each open set U containing $C - D$ and for each positive number ϵ, there is a closed map h satisfying

(1) $h = \text{Id}$ on $E^3 - U$,
(2) $h(C) \subset D$,
(3) $h(x) = h(y)$ only if $x = y$ or $\{x, y\} \subset C$ for $\{x, y\} \subset E^3$, and
(4) $\text{diam } h(g) < \text{diam } g + \epsilon$ for each $g \in G$.

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