

COMPACT p -ADIC SEMIGROUPS

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Introduction. A topological semigroup is p -adic, where p is a positive integer, if it contains a dense homomorphic image of the additive semigroup of positive p -adic rational numbers. Compact p -adic semigroups are the fundamental building blocks of compact p -divisible semigroups. The purpose of this paper is to construct a universal compact p -adic semigroup in the sense that each compact p -adic semigroup is a continuous homomorphic image of this one. Some of the techniques used in this construction are similar to those employed in [4]. Related material appears in [5], [7] and [9].

The author wishes to express his appreciation to Professor R. J. Koch for his contributions to some of the results of this paper and to the referee for his suggestions for modifications.

The *Bohr compactification* of a topological semigroup S is a pair $(B(S), b_S)$ such that

- (i) $B(S)$ is a compact semigroup,
- (ii) $b_S: S \rightarrow B(S)$ is a continuous homomorphism such that $b_S(S)$ is dense in $B(S)$, and
- (iii) if $f: S \rightarrow T$ is a continuous homomorphism of S into a compact semigroup T , then there exists a continuous homomorphism $h: B(S) \rightarrow T$ such that $h \circ b_S = f$.

For a topological semigroup S we will hereafter use $b_S: S \rightarrow B(S)$ to denote the Bohr compactification of S . The existence and uniqueness of the Bohr compactification is a consequence of the adjoint functor theorem [10]. Related results can be found in [1].

If A and B are groups, then $\text{Hom}(A, B)$ denotes the group of homomorphisms of A into B .

We will use \prod to denote cartesian semigroup (or group) products and \oplus to denote direct sums of discrete groups. If G is a locally compact abelian group, then G^\wedge denotes the character group of G and G_d denotes the group G given the discrete topology.

Notation. The following notation will be used throughout this paper.

1. N is the set of all positive integers;
2. p is an element of N ($p \geq 2$);
3. G_p is the discrete additive group of all p -adic rationals;
4. Q_p is the semigroup of positive elements of G_p ;
5. Ω_p is the compact group of p -adic integers, where the defining sequence is (p, p, \dots) [2; 402];

Received March 17, 1970. Revision received April 17, 1970.