ON THE BERGMAN METRICS AND THEIR INDICATRIXES

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1. **Introduction.** When applying the method of the kernel function we present various quantities with the invariant metric ds_D^2 as rational functions of minimal values of $\|f\|^2$, where f belongs to different subsets of $\mathcal{L}^2H(D)$. These minima values $\lambda_D^{\cdot\cdot}$ are monotone functions of the domain D. Comparing these values with corresponding values for domains for which the kernel function is known explicitly one obtains various distortion theorems. This method, called the *method of the minimum integral,* is generalized in the way of introducing a convenient complete system of orthonormal functions $\{\psi^k\}$. Using the $\{\psi^k\}$ we obtain a new expression for the Bergman metric and some other related metrics. These representations enable us to obtain various new distortion theorems and geometrical interpretations. In particular we give new proofs for the fact that the Bergman metric is positive definite.

Let D be a bounded domain in C^n and consider the Bergman kernel function $K_D(z, \bar{t})$ of the domain D, $z, t \in D$. $K_D(z, \bar{t})$ as a function of z is the *reproducing kernel* of the separable Hilbert space $\mathcal{L}^2H(D)$ which consists of all analytic functions $f(z)$, $z \in D$, for which

$$\|f\|^2 = (f, f) = \int_D |f(z)|^2 \, d\Omega_z < \infty. \tag{1.1}$$

Here

$$z_k = x_k + iy_k, \qquad k = 1, 2, \cdots, n, \qquad d\Omega_z = \prod_{k=1}^{n} (dx_k \, dy_k). \tag{1.2}$$

For any complete orthonormal system $\{\varphi^\nu\}_{\nu=1}^{\infty}$ of $\mathcal{L}^2H(D)$ we have the bilinear expansion (see [1], [2] and [4])

$$K_D(z, \bar{t}) = \sum_{\nu=1}^{\infty} \varphi^\nu(z)\overline{\varphi^\nu(t)}. \tag{1.3}$$

The infinite sum on the right is independent of the particular choice of the complete orthonormal sequence and converges uniformly in each compact subset of D. Further $K_D(z, \bar{t})$ is analytic in $\bar{t}$ since

$$K_D(z, \bar{t}) = \overline{K_D(t, \bar{z})}. \tag{1.4}$$

Also the functions

$$K_{k_1 \cdots k_n \bar{\mu}_1 \cdots \bar{\mu}_n}(z, \bar{t}) = \frac{\partial^{k_1 + \cdots + k_n + \mu_1 + \cdots + \mu_n} K_D(z, \bar{t})}{\partial z_1^{k_1} \cdots \partial z_n^{k_n} \, \partial \bar{t}_1^{\mu_1} \cdots \partial \bar{t}_n^{\mu_n}} \tag{1.5}$$

Received February 23, 1970. Revision received May 24, 1970. This work was partially supported by Contract F 4620-69-CO106 at Stanford University.