NOTE ON THE NUMBERS OF JORDAN AND WARD

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1. We define the Stirling numbers of the first kind by means of

$$x(x+1)\cdots(x+n-1) = \sum_{k=0}^{n} S_1(n,k)x^k \tag{1.1}$$

and those of the second kind by

$$x^n = \sum_{k=0}^{n} S(n,k)x(x-1)\cdots(x-k+1). \tag{1.2}$$

It is familiar that

$$S_1(n+1,k) = S_1(n,k-1) + nS_1(n,k) \tag{1.3}$$

and

$$S(n+1,k) = S(n,k-1) + kS(n,k). \tag{1.4}$$

Jordan [5; Chapter 4] and Ward [7] have introduced certain arrays of numbers related to $S(n, k)$ and $S_1(n, k)$. We shall not use the notation of these writers. In the first place we put

$$S_1(n, n-k) = \sum_{j=0}^{k-1} S_1'(k,j)\binom{n}{2k-j} \qquad (k > 0) \tag{1.5}$$

for the numbers of the first kind. Similarly, for the numbers of the second kind we put

$$S(n, n-k) = \sum_{j=0}^{k-1} S'(k,j)\binom{n}{2k-j} \qquad (k > 0). \tag{1.6}$$

The coefficients $S'(k, j)$ satisfy the recurrence

$$S'(k+1,j) = (k-j+1)S'(k,j-1) + (2k-j+1)S'(k,j), \tag{1.7}$$

while $S_1'(k, j)$ satisfy

$$S_1'(k+1,j) = (2k-j+1)(S_1'(k,j-1) + S_1'(k,j)). \tag{1.8}$$

The coefficients $S'(k, j)$ have occurred recently in a rather unexpected connection [1]. Put

$$e^{nz} = \sum_{r=0}^{n} \frac{(nz)^r}{n!} + \frac{(nz)^n}{n!} S_n(z).$$

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