PRODUCTS OF MINIMAL URYSOHN SPACES

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A topological space is said to be *Urysohn* if every pair of distinct points have disjoint closed neighborhoods. A Urysohn space $(X, \mathfrak{I})$ is said to be *minimal Urysohn* if there does not exist a Urysohn topology on X properly contained in $\mathfrak{I}$.

In this paper we provide a negative answer to a question of C. T. Scarborough: Is the product of minimal Urysohn spaces minimal Urysohn? We also give an example of a first countable minimal Urysohn space that is neither regular, of second category, nor minimal first countable Hausdorff.

1. A characterization theorem. A filter base on a space X is said to be *open* if the sets belonging to it are open subsets of X. An open filter base $\mathfrak{F}$ is said to be a *U-filter* [4] provided that for every $x \in X$, if x is not an adherent point of $\mathfrak{F}$, then there are a neighborhood V of x and a set $F \in \mathfrak{F}$ for which $\bar{V} \cap \bar{F} = \emptyset$.

We shall call an open filter base $\mathfrak{F}$ on a space X an *almost U-filter* if there exists $p \in X$ so that for every $x \in X - \{p\}$, if x is not an adherent point of $\mathfrak{F}$, then there are a neighborhood V of x and a set $F \in \mathfrak{F}$ for which $\bar{V} \cap \bar{F} = \emptyset$.

Recall that a space $(X, \mathfrak{I})$ is *semiregular* if $\{\overset{\circ}{\bar{T}} \mid T \in \mathfrak{I}\}$ is a base for $\mathfrak{I}$.

Theorem 1. *Let X be a Urysohn space. The following are equivalent.*

(i) *X is minimal Urysohn.*
(ii) *Every U-filter on X with a unique adherent point is convergent.*
(iii) *X is semiregular and every almost U-filter on X has an adherent point.*

The observation that (i) and (ii) are equivalent is due to C. T. Scarborough [4] (for another characterization theorem, see [2]), and it is also noted in [4] that a minimal Urysohn space is semiregular.

The proof of the equivalence of (ii) and (iii) is routine, especially if one uses the following obvious fact: If $V = \overset{\circ}{\bar{V}}$ and if F is an open set such that $V \not\supset F$, then $F - \bar{V} \neq \emptyset$.

2. The examples. In [2] Herrlich constructs a noncompact minimal Urysohn space X. We shall construct two minimal Urysohn spaces Y and Z and, in the following section, prove that $X \times Y \times Z$ is not minimal Urysohn (or U-closed [4]).

Description of X. Let $\Omega(\omega)$ be the first uncountable (countable) ordinal,

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