SYMMETRIC INVOLUTIONS OVER FIELDS OF CHARACTERISTIC 2

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1. Introduction. An $n \times n$ matrix over any field K is said to be an *involution* if $A^2 = I_n$ over K. Throughout this paper, field K, unless otherwise specified, will have characteristic two. If K is a finite field of order $q = 2^t$, it will be denoted by F_q. The $k \times k$ identity matrix will be denoted by I_k.

The $n \times n$ involution A over K has been said by Hodges [5] to have *signature s*, for some $s = 0, 1, 2, \cdots, [n/2]$, if A is similar to the direct sum matrix

$$D(I_{n-2s}, H_1, H_2, \cdots, H_s), \tag{1.1}$$

where $[n/2]$ is the greatest integer $\leq n/2$ and where for each i, $i = 1, 2, \cdots, s$,

$$H_i = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

Any two of the following imply the third, where A is any $n \times n$ matrix over any field K.

$$A \text{ is an involution.} \tag{1.2}$$

$$A \text{ is symmetric.} \tag{1.3}$$

$$A \text{ is orthogonal.} \tag{1.4}$$

Fulton [4] has characterized the $n \times n$ symmetric involutions of signature s over fields of characteristic $\neq 2$ and over the ring of integers Z_{p^t} modulo p^t, p odd. In the same paper, the $n \times n$ symmetric involutions over any finite field of characteristic $\neq 2$ are enumerated by signature.

In this paper, the $n \times n$ symmetric involutions over a field K of characteristic two are characterized and are enumerated in the case that $K = F_q$.

Levine and Nahikian [6] have characterized the $n \times n$ involutions over K by signature in the following theorem.

THEOREM 1.1. *A is an $n \times n$ involutory matrix of signature s over K if and only if A can be decomposed as $I_n + QP$, where each of Q and P^T is $n \times s$ of rank s over K and where $PQ = 0$.*

In §2, Theorem 1.1 is used to prove Theorem 1.2 in which the $n \times n$ symmetric involutions over K are characterized.

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