# G-SPACES MOD F AND H-SPACES MOD F

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In a series of papers [3], [4], and [5] D. H. Gottlieb has introduced and studied the evaluation subgroup $G_m(X)$ of $\pi_m(X)$; $G_m(X)$ is defined to be the set of all $\beta \varepsilon \pi_m(X)$ for which there is a representative $b$ of $\beta$ and a map $F: X \times S^m \to X$ of type (Id, $b$); that is, $F \mid X = \text{Id}$, the identity mapping of $X$, and $F \mid S^m = b$. A space $X$ satisfying $G_m(X) = \pi_m(X)$ for all $m$ is called a $G$-space; evidently $H$-spaces are $G$-spaces, however, there are $G$-spaces which are not $H$-spaces [5] (see also [6] and [12]).

Let $C$ denote a Serre class of abelian groups. We call a space $X$ a $G$-space mod $C$ if $\pi_m(X)/G_m(X) \varepsilon C$ for all $m$. In this note we study $G$-spaces mod $C$. Our main concern is with the class $F$ of finite abelian groups and our main result is the following theorem.

THEOREM 1. *Let $X$ be a* 1*-connected finite* CW*-complex. Then $X$ is a $G$-space* mod $F$ *if and only if $X$ is an $H$-space* mod $F$.

Theorem 1 follows from the slightly more general Propositions 3 and 5 which are stated and proved in §§2 and 3 respectively. For 1-connected spaces, Theorem 1 generalizes to $G$-spaces mod $F$ and improves some of the results of [4]. For example, Theorem 7–3 of [4] states that if $X$ is a finite CW-complex and a $G$-space, then the Euler characteristic of $X$ is zero.

In §4 we give two applications of Theorem 1. In particular, we show that an $H$-space mod $F$ admits a "multiplication" having a one-sided unit. Recall that a space $X$ is an $H$-space mod $F$ if there is a map $X \times X \to X$ whose restriction to each factor of its domain induces an $F$-isomorphism of integral homology groups in each dimension.

**1. Preliminaries.** In this note we only consider based spaces and we require all maps and homotopies to preserve base points. In fact, we will restrict our attention to the category $T$ whose objects $X$ are spaces having the homotopy type of a 1-connected countable CW-complex such that *, the base point of $X$, is closed in $X$ and $(X, *)$ has the homotopy extension property (HEP). In view of the following lemmas it is apparent that $T$ will be a convenient category for the study of the evaluation subgroup $G_m(X)$ of $\pi_m(X)$.

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