MONTGOMERY-SAMELSON COVERINGS ON MANIFOLDS

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1. In this note we generalize a theorem concerning the homology of the branch set of a Montgomery–Samelson (M–S) covering [6, Theorem 2] to Čech homology. We apply this result by studying regular M–S coverings defined on complex projective space (a regular branched covering is the orbit map of the action of a finite solvable group). Throughout this note, f denotes a map from the compact, connected n-manifold M onto the n-manifold N which is a local homeomorphism except at points of the *branch set*, which we denote by B_f. We assume that $\dim B_f \leq n - 2$. This implies that $f \mid (M - f^{-1}fB_f)$ is a d-to-1 covering map [1, Theorem 2.4]. We call such a map a *branched covering.* If $f^{-1}fB_f = B_f$ and $f \mid B_f$ is a homeomorphism, we say that f is an *M–S covering.* Throughout, the letter H refers either to Čech or to simplicial homology as the case may be, unless the letter has a superscript.

2. The Dimension Sum Theorem. Let M and N be connected compact metric spaces. Let N have a basis $\mathcal{B}$ of open sets such that the non-empty finite intersections of basis elements are arcwise connected. Let $f\colon M \to N$ be a surjective, light, open map such that $f^{-1}(y)$ has at most d elements for any y in N. Let B_f be the set at which f fails to be a local homeomorphism, assume that $f(B_f)$ is the set of points that have less than d inverses and that $f/f^{-1}f(B_f)$ is a homeomorphism. Let $f/(M - f^{-1}f(B_f))$ be a d-to-one covering map and assume that $f^{-1}f(B_f)$ does not separate M locally anywhere. These conditions are satisfied in case M and N are orientable n-manifolds, f is light open, and $\dim B_f \leq n - 2$ [2, 2.4].

The idea of the following construction is to find a covering U of M that maps onto a covering V of N in such a way that the elements of U stack up neatly above those of V and behave reasonably at B_f.

Designate the cardinality of a set A by $\#A$. Let $q \in N$ and let $f^{-1}(q) = \{q_1, \cdots, q_k\}$, $k \leq d$. Let V_q be an element of $\mathcal{B}$ containing q. Let $U_1, \cdots, U_r$ be the components of $f^{-1}(V_q)$. By the arc lifting theorem [9, p. 186, Theorem 2.1], each U_i contains at least one q_j and $f(U_i) = V_q$, thus $r \leq k$.

Lemma 1. *If $y \in V_q - f(B_f)$, then $\#(f^{-1}(y) \cap U_i)$ is independent of y; and, if $r = d$, then f/U_i is a homeomorphism.*

Proof. Let $z \in V_q$ and join z to q by an arc. This arc lifts to one from some point $q_j \in f^{-1}(q) \cap U_j$ to a point, say z_j, of $f^{-1}(z)$. Since U_i is a component of $f^{-1}(V_q)$, U_i contains z_j. Since $f^{-1}f(B_f)$ does not separate U_j, it follows that

Received November 1, 1969. This author was supported in part by Army Research Office-Durham grant ARO-D-31-124-G892, S2.