ORTHOGONALITY RELATIONS FOR A CLASS OF BRENKE POLYNOMIALS

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1. The Brenke type generating functions are the polynomial generating functions of the form

$$A(w)B(xw) = \sum_{n=0}^{\infty} P_n(x)w^n \tag{1}$$

where

$$A(w) = \sum_{k=0}^{\infty} a_k w^k, \qquad a_0 \neq 0$$

$$B(w) = \sum_{k=0}^{\infty} b_k w^k, \qquad b_k \neq 0 \quad \text{for} \quad k \geq 0.$$

Thus the corresponding Brenke polynomials are

$$P_n(x) = \sum_{k=0}^{n} a_{n-k} b_k x^k. \tag{2}$$

In [1] we determined all pairs $(A(w), B(w))$ for which $\{P_n(x)\}$ is an orthogonal polynomial sequence (OPS). The resulting classes of OPS consisted of: (i) Laguerre polynomials; (ii) Szegö's generalized Hermite polynomials; (iii) a class of generalized Stieltjes-Wigert polynomials; (iv) a class of polynomials first encountered by H. S. Wall; (v) a symmetric OPS related to the Wall polynomials; (vi) two related sets of orthogonal q-polynomials studied by W. A. Al-Salam and L. Carlitz; (vii) an OPS for which we were unable to find explicit orthogonality relations. (We refer the reader to [1] for more explicit descriptions of these polynomials as well as references.)

The polynomials in (vii) are generated by (1) with

$$\begin{gathered} a_{2m} = a_{2m+1} = \frac{(-1)^m q^{m(m+1)/2}}{[q]_m} \\ b_{2m} = \frac{1}{[q]_m [b]_m}, \qquad b_{2m+1} = \frac{1}{[q]_m [b]_{m+1}} \\ (q \neq 0, 1;\, b \neq 0;\, bq^m \neq 1 \quad \text{for} \quad m = 0, 1, 2, \cdots), \end{gathered} \tag{3}$$

where we have used the notation

$$[a]_0 = 1, \quad [a]_m = (1-a)(1-aq)\cdots(1-aq^{m-1}) (m \geq 1).$$

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