# THE RELATION OF BREADTH AND CODIMENSION IN TOPOLOGICAL SEMILATTICES, II

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A problem posed by E. Dyer and A. Shields in [5] is whether the dimension and breadth are equal in a compact connected metrizable distributive topological lattice. L. W. Anderson [1 and 2] has shown that if $L$ is a locally compact, connected distributive lattice, then the codimension bounds the breadth. The author extended these results in [8] to show that if $S$ is a locally compact, chain-wise connected topological semilattice, then the breadth does not exceed the codimension by more than one; furthermore, if $S$ is directed upward then the breadth is less than or equal to the codimension. As a corollary Anderson's result holds without distributivity.

In the other direction T. H. Choe [3] has recently shown that if $L$ is a locally compact connected distributive topological lattice of inductive dimension $n$ and if the set of points at which $L$ has dimension $n$ has non-void interior, then breadth bounds dimension. The purpose of this paper is to derive the more general result that the breadth bounds the codimension in any locally compact topological semilattice. Thus in a locally compact, connected lattice the breadth and codimension are equal.

**1. Preliminaries.** A *topological semilattice* is a Hausdorff topological space endowed with a partial order for which every two elements $x$, $y$ have a greatest lower bound denoted $x \wedge y$ and the function $(x, y) \rightarrow x \wedge y$ is continuous. The breadth of a topological semilattice is the smallest integer $b$ such that any meet $x_1 \wedge \cdots \wedge x_n (n > b)$ is always a meet of a subset of $b$ of the $x_i$. In general, a semilattice need not have finite breadth. A topological semilattice has *small semilattices* if there exist a basis of neighborhoods which are subsemilattices. Semilattices with small semilattices have been studied in [9]. A discussion of codimension may be found in [4].

THEOREM 1.1. *Let $S$ be a topological semilattice with finite breadth. Then $S$ has small semilattices.*

*Proof.* Suppose that $S$ has breadth $n$, and $x \in U$ an open subset of $S$. By continuity of the meet operation, there exists an open set $V$ containing $x$ such that $V^n \subset U$. We show that the subsemilattice $T$ generated by $V$ is contained in $V^n$.

Let $z \in T$. Then there exists $\{v_i\}_{i=1}^m \subset V$ such that $z = \wedge_{i=1}^m v_i$. Since $S$ has breadth $n$, there exists a subset of $n$ of the $v_i$ with meet equal to $z$. Hence

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