FINITE PRODUCTS OF WALLMAN SPACES

By Frank Kost

Introduction. In 1938 H. Wallman [5] associated with a T_1-space X a compact T_1-space Y which contains a dense copy of X. In this construction the points of Y are the ultrafilters from the lattice of closed subsets of X. O. Frink [2] characterized complete regularity in T_1-spaces as those spaces that possess a normal base for their closed sets. In the proof of sufficiency he constructs a Hausdorff compactification $\omega(Z)$ of X by considering the ultrafilters from the normal base Z. As this construction was utilized by Wallman, Frink calls a compactification obtained in this way a *Wallman-type compactification,* or a *Wallman space.* Frink then offered the conjecture that every compactification is Wallman-type. This question is unsettled. Some partial solutions are known. E. F. Steiner [4] has given sufficient conditions on a compact space in order that it be a Wallman-type compactification of each of its dense subspaces. From the work of R. M. Brooks [1] we have that all compactifications of discrete space obtained by "adding" no more than a countable number of points are Wallman-type. So is the one-point compactification of a locally compact space. In the sequel we offer a partial solution to the conjecture by showing that finite products of Wallman spaces are again Wallman spaces.

Definitions. Call Y a *compactification* of the space X if Y is a compact Hausdorff space and X is dense in Y (or a copy of X is dense in Y). A family $\mathfrak{F}$ of closed sets of a space X is said to be

i) a *base* for the closed sets if for $x \notin A$, A closed there is $F \in \mathfrak{F}$ such that $x \notin F$ and $A \subset F$.
ii) *ring of sets* if $\mathfrak{F}$ is closed under formation of finite unions and intersections.
iii) *disjunctive* if given $x \notin A$, A closed there is $F \in \mathfrak{F}$ with $x \in F$ and $F \cap A = \emptyset$.
iv) a *normal family* if given $F_1, F_2 \in \mathfrak{F}$, $F_1 \cap F_2 = \emptyset$, there is $H_1, H_2 \in \mathfrak{F}$ such that $F_1 \subset X\backslash H_1$, $F_2 \subset X\backslash H_2$ and $(X\backslash H_1) \cap (X\backslash H_2) = \emptyset$. If $\mathfrak{F}$ has properties i $\rightarrow$ iv above, $\mathfrak{F}$ is called a *normal base* for the closed sets of X. The collection of all closed sets of a T_1-space satisfies i $\rightarrow$ iii and is a normal base if and only if X is a normal space. For X a discrete space we have $\{F \subset X: F \text{ finite or } X\backslash F \text{ finite}\}$ as a normal base. Also the collection of zero-sets of continuous real-valued functions on X is a normal base in case X is completely regular-T_1 ($T_{3\frac{1}{2}}$).

For $\mathfrak{F}$ a ring of closed sets of X that is a disjunctive base for the closed sets of X we set $\omega(\mathfrak{F}) \equiv \{\mathfrak{A}: \mathfrak{A} \subset \mathfrak{F},\ \mathfrak{A} \text{ an ultrafilter}\}$. The ultrafilters from $\mathfrak{F}$ with

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