DEFORMATIONS OF FUCHSIAN GROUPS, II

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1. Introduction and summary of results. This paper is a continuation of the study begun in [7] and [8]. Let G be a Fuchsian group acting on Δ or U, where Δ is the unit disc and U is the upper half plane. A *deformation* of G is a pair (χ, f) where (i) χ is a homomorphism of G into the group of Möbius transformations, and (ii) f is a meromorphic, local homeomorphism of Δ or U onto an open subset D of the complex sphere, $\mathbf{C} \cup \{\infty\}$, such that

$$f \circ A = \chi(A) \circ f \quad \text{for every} \quad A \in G. \tag{1.1}$$

The set of deformations admits a natural equivalence relation; two deformations (χ_1, f_1) and (χ_2, f_2) of G are called *equivalent* if there is a Möbius transformation B such that

$$f_2 = B \circ f_1 \quad \text{and} \quad \chi_2(A) = B \circ \chi_1(A) \circ B^{-1} \quad \text{for all} \quad A \in G. \tag{1.2}$$

The set of equivalence classes of deformations of G is in a natural one-to-one correspondence with $Q(G)$, the vector space of automorphic forms of weight (-4). The deformation (χ, f) corresponds to the point $\varphi = Sf \in Q(G)$, where S is the Schwarzian differential operator. We shall often denote (χ, f) by $(\chi_\varphi, f_\varphi)$. We shall consider only those deformations $(\chi_\varphi, f_\varphi)$ for which $\varphi \in B(G)$, where $B(G)$ is a Banach subspace of $Q(G)$ consisting of the bounded automorphic forms of weight (-4). If G is finitely generated of the first kind, then $B(G)$ is finite dimensional, but $Q(G)$ in general is not. We assume throughout this paper (except in the corollary to Theorem 2) that the dimension of $B(G)$ is non-zero. We will denote the norm in $B(G)$ by $\|\cdot\cdot\|$. Abbreviate $B(\{1\})$ by B.

Our main result is

Theorem 1. *Let (χ, f) be a deformation of a Fuchsian group G. Assume that G is finitely generated and of the first kind, and that $Sf \in B(G)$. Then the following are equivalent*:

(a) *$\chi(G)$ acts discontinuously on D,*
(b) *f is a covering map, and*
(c) *$D \neq \mathbf{C} \cup \{\infty\}$.*

Furthermore, if the above conditions are satisfied, then D is an invariant component of the region of discontinuity of the Kleinian group $\chi(G)$.

In [7] we proved Theorem 1 under the assumption that G is the covering group of a compact Riemann surface. Thus our main result is a generalization of our previously published Theorem. As in [7], we use the word "covering" to mean "unbranched, unramified, covering."

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