ON THE CONTINUITY OF CERTAIN HOMOMORPHISMS OF COMPACT SEMIGROUPS

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We shall say that a topological semigroup has the *van der Waerden property* if each algebraic automorphism on it is continuous. The purpose of this note is to consider compact semigroups which enjoy the van der Waerden property. (One could also use the term "Cartan property" since it can be inferred from [5]. However, it is not therein explicit.) According to the continuity principle of van der Waerden [1], an almost periodic function defined on a compact semi-simple Lie group G is necessarily continuous. It follows that any algebraic homomorphism of G into an arbitrary compact group is continuous and so, compact semi-simple Lie groups have the van der Waerden property. On the other hand, if G is a finite dimensional compact connected group then $G = (L \times H)/F$ where L is a semi-simple Lie group, H is a compact connected abelian group and F is a finite group. Now, if H is nontrivial, then it is algebraically the cartesian product of groups $Z(p^\infty)$ and additive real numbers. Letting R denote the real numbers we may write $H = R \times W$. Now, by using a Hamel basis one can obtain a discontinuous automorphism of R leaving fixed any finite number of preassigned points. One then extends this to a discontinuous automorphism of H and then to $L \times H$ leaving F fixed pointwise and so induces a discontinuous automorphism on $L \times H/F = G$. Thus we have

Proposition 1. *A finite dimensional compact connected group is a semi-simple Lie group if, and only if, it has the van der Waerden property.*

We now recall [3] that the Bohr compactification $\hat{G}$ of a semigroup G is that compact semigroup $\hat{G}$ such that any diagram

$$\begin{tikzcd} \hat{G} \\ G \arrow[u, "\sigma"] \arrow[r, "\xi"'] & T \end{tikzcd}$$

where ξ is a dense continuous representation into the compact semigroup T, completes to

$$\begin{tikzcd} \hat{G} \arrow[dr, "\hat{\xi}"] \\ G \arrow[u, "\sigma"] \arrow[r, "\xi"'] & T \end{tikzcd}$$

where $\hat{\xi}$ is a continuous epimorphism. (For certain semi-groups see [4].)

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