SPACES OF CERTAIN NON-ALTERNATING MAPPINGS

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Introduction. The concept of a non-alternating mapping is a natural generalization of the notion of a monotone mapping. This concept was introduced by G. T. Whyburn [5] in 1934. Some idea of the usefulness of non-alternating mappings may be realized through reading references [4]; [5]; [6]; [7]; [8]. However, a wealth of knowledge about non-alternating mappings lies untouched. Many important applications remain to be discovered.

We wish to consider briefly the space of all non-alternating mappings of a compact metric continuum X (in particular, a Peano continuum) onto the interval $[0, 1] = I$. Such a space is topologically complete. Furthermore, one of its subspaces, the space of all open non-alternating mappings of X onto I is also topologically complete. For certain continua, the spaces of all light open non-alternating mappings onto I have certain local connectivity properties. We show that our results along with other assumptions yield the existence of light open mappings from a 1-dimensional Peano continuum onto a 2-cell.

Definitions. A mapping f from a connected space X onto a connected space Y is said to be *non-alternating* iff for each p and q in Y, $f^{-1}(p)$ fails to separate $f^{-1}(q)$ in X. A mapping is open iff for each open set U in X, $f(U)$ is open in X [if the mapping is not onto Y, then $f(U)$ is open relative to $f(X)$]. And, it is *light* iff $f^{-1}f(x)$ is totally disconnected (degenerate components) for each x in X.

Spaces of non-alternating and non-alternating open mappings. Theorem 2 below illustrates a property peculiar to non-alternating mappings onto an interval. A sequence of monotone mappings (even homeomorphisms) of a compact metric space into itself which converges uniformly may converge to a mapping which is not monotone [9]. However, a sequence of non-alternating mappings of X onto I which converges uniformly must converge to a non-alternating mapping.

First, we state Theorem 1 without proof. We use it in a proof of Theorem 2.

Theorem 1. *Suppose that n is a non-alternating mapping of a compact metric continuum X onto I. Then $N = \{n^{-1}(\rho) \mid \rho \in I\}$ is a non-separated collection. Furthermore, both $n^{-1}(0)$ and $n^{-1}(1)$ fail to separate X. But, $n^{-1}(p)$ for $0 < p < 1$ separates X uniquely.*

A proof of Theorem 1 follows from the results in a paper of Whyburn [4].

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