CONNECTIVITY MAPS AND ALMOST CONTINUOUS FUNCTIONS

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1. Introduction. In [3] Stallings proves certain results about the composite of non-continuous functions and poses questions about other possibilities. Throughout this paper I is used to denote the closed interval [0, 1]. In this paper we are primarily concerned with another way of combining non-continuous functions and in particular we show that if Y is a space, a, b, and c are numbers such that $a < c < b$, and $F : [a, b] \times I \to Y$ is a function such that $F|_{[a,c]\times I}$ and $F|_{[c,b]\times I}$ are connectivity maps and $F|_{\{c\}\times I}$ is continuous, then F is a connectivity map. To get a similar theorem about almost continuous functions, we need to assume that Y is a locally convex subset of a real linear topological space.

In [1] Hildebrand and Sanderson define the concept of a connectivity retract and get results, some of which parallel results known about continuous retracts. We define the concept of connectivity homotopy and relate connectivity retracts to connectivity homotopy. We also define the concepts of an almost continuous retract and almost continuous homotopy and get results which parallel the results in the case of connectivity maps.

The reader is referred to [1], [2], and [3] for definitions and notation not covered in this paper. The author wishes to express appreciation to C. Wayne Patty for useful conversations.

2. Preliminary result. In [3] Stallings shows that if f is a connectivity map and g is continuous, then gf is a connectivity map and gives an example where f is continuous, g is a connectivity map, and gf is not a connectivity map. However, the following theorem shows that when f is a homeomorphism, gf is a connectivity map. The corresponding result about almost continuous functions is also true.

Theorem 2.1. *If $f : (X, R) \to (Y, S)$ is a homeomorphism and $g : (Y, S) \to (Z, T)$ is a connectivity (almost continuous) function, then $gf : (X, R) \to (Z, T)$ is a connectivity (resp. almost continuous) function.*

Proof. Let $\alpha = \{g^{-1}(U) \mid U \in T\}$, $\beta = \{(gf)^{-1}(U) \mid U \in T\}$, and $\gamma = \{f^{-1}(A) \mid A \in \alpha\}$. Let S' be the topology generated by S and α and let R' be the topology generated by R and β. It is a simple calculation to see that $\beta = \gamma$ and thus, R' is also the topology generated by R and γ. Since $f : (X, R) \to (Y, S)$ is a homeomorphism, $f : (X, R') \to (Y, S')$ is a homeomorphism. Let C be a connected subset of (X, R). Then $f(C)$ is a connected subset of (Y, S).

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