EXTENDING MAPS AND DIMENSION THEORY

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The problems concerned with the extension of continuous functions have been combined with dimension theory to yield the following general question: When can continuous functions on a metric space X be extended to a completion or compactification of X which has the same dimension as X? With the use of results of W. Hurewicz [2], one solution was provided by A. B. Forge [1] in a theorem which can be restated as follows (throughout this paper we denote by dim the covering dimension of Lebesgue):

THEOREM. *Suppose X is a separable metric space, and for each $i = 1, 2, \cdots$ let f_i be a continuous function from X into the closed unit interval. Then there exists a compactification X^* of X with* $\dim X^* = \dim X$ *to which each f_i can be continuously extended.*

K. Nagami [3] has succeeded in generalizing this theorem in two directions: first, by replacing the unit interval with any compact metric space, and second, by developing an analogue for arbitrary (not necessarily separable) metric spaces. In this paper we shall combine the work of Nagami with previous work of the author [5], [7] to further generalize these results.

The following definitions will be needed.

DEFINITION. Let (X, ρ) be a metric space, and let $Y \subset X$, $\dim Y = n$, and ρ_Y be the induced metric on Y. Define $S_\alpha(x \mid Y) = \{y \in Y: \rho_Y(y, x) < \alpha\}$. Then ρ *has Property A on* Y [cf. 7] iff there exists $\delta > 0$ such that for every positive $\epsilon < \delta$ and every $x \in Y$,

$$\rho_Y(S_{\epsilon/2}(x \mid Y), y_i) < \epsilon \quad (i = 1, \cdots, n+2)$$

imply

$$\rho_Y(y_i, y_j) < \epsilon \quad \text{for some} \quad i, j \quad \text{with} \quad i \neq j.$$

We say ρ *is dimension-lowering on* Y iff for every $\epsilon > 0$ and every $x \in X$, $\dim [Y \cap Bdry\, S_\epsilon(x)] < n$.

A central role will be played by the following result; for later work in this paper we shall be principally interested in conditions i) and iv).

THEOREM 1. *Let X be a metric space with a bounded metric d, and for each $k = 1, 2, \cdots$ let X_k be a nonvoid closed finite-dimensional subset of X. Then there exists an equivalent metric ρ for X such that*

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