A NOTE ON RINGS WITH NOETHERIAN SPECTRUM

By William Heinzer

1. Introduction. Let R be a commutative ring with identity, and let A be an ideal of R. Following the notation of Ohm and Pendleton in [4], A is called a J-radical ideal if A is an intersection of maximal ideals; the J-components of A are the minimal members of the family of J-radical prime ideals containing A; R has JFC means that each ideal of R has only a finite number of J-components; and R has JN means that the J-radical ideals of R satisfy the ascending chain condition. Proving the easily established fact that R having JFC implies R has JN enables us to give some sufficient conditions that JN be preserved by an R-algebra. We obtain as a corollary a new proof for the result of Ohm and Pendleton that R having JN implies that a finite R-algebra has JN. Again following [4], we say that R has FC if each ideal of R has only a finite number of minimal prime divisors. We investigate the question mentioned in [4] of whether R having FC and R' a finite R-algebra implies R' has FC. Difficulty here seems to arise from the fact that R having FC does not imply a noetherian property for the prime spectrum of R. We show in some special cases that R having FC implies that R' has FC (e.g., when R is the fixed ring of a finite group of automorphisms of R'), however, we are not able to settle the general question.

All our rings are assumed to be commutative with identity. If R and R' are rings and $\varphi : R \to R'$ endows R' with the structure of an R-algebra, we assume that $\varphi(1)$ is the identity element of R'.

2. Topological lemmas. Following a suggestion by Jack Ohm we will state our principal lemmas in topological terms, thus making them applicable both to the question on finite J-components and the question on FC. We first recall several definitions found in [1; 119–121]. A topological space X is called *irreducible* if the intersection of two nonempty open subsets of X is again nonempty; and the *irreducible components* of X are the maximal irreducible subsets of X. One then applies these concepts to a subset B of X by giving B the relative topology. The following is immediate from the definition of an irreducible set.

Fact 1. Suppose that $\{P_i\}_{i=1}^n$ are the irreducible components of a subset B of X, and that $T_i \subseteq P_i$ are closed sets such that $\bigcup_{i=1}^n T_i = B$. Then $T_i = P_i$ for each i.

We will use the notation Cl (A) to denote the closure of a subset A of a topological space X.

Lemma 2. *Let X' and X be topological spaces and let $f : X' \to X$ be a continuous*

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