CONTINUA THAT CONTAIN ONLY DEGENERATE CONTINUOUS IMAGES OF PLANE CONTINUA

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1. Introduction. Anderson and Choquet have given an example [1] of a plane continuum (compact, connected metric space), M, no two of whose non-degenerate subcontinua are homeomorphic. It has been observed before that M is a tree-like plane continuum that contains only degenerate chainable continua. The question that led to the results here was the following: "Does the cartesian product $M \times M$ contain a non-degenerate chainable continuum?" This question is not entirely settled, for there are many continua which can be said to be defined by the process which Anderson and Choquet describe. Nevertheless, we show (Theorem 1) that one such continuum contains only degenerate continuous images of chainable continua, a property that is inherited by any product space all of whose factors are homeomorphic to it (see Lemma 4). Moreover, there is a plane continuum which contains only degenerate continuous images of plane continua no subcontinuum of which separates the plane (Theorem 2). Similarly, we show that the one-dimensional continua M_1 and M_2 recently obtained by H. Cook [2, p. 248] contain only degenerate continuous images of plane continua; so that for each n, $M_1 \times M_1 \times \cdots \times M_1$ (n factors) is an n-dimensional continuum with the same property.

The distance between two points A and B is denoted by $d(A, B)$; the diameter of a point set H by $D(H)$. For inverse limits we use the conventions of [1], except that $\{X_i, f_i\}$ will be used to denote an inverse limit sequence, and π_n to denote the projection mapping from the limit onto X_n. If X and Y are continua, the transformation f from X onto Y is said to be an *atomic mapping* if and only if f is continuous, monotone, and if K is a subcontinuum of X and $f(K)$ is non-degenerate, then $K = f^{-1}f(K)$. If, in addition, only finitely many points of Y have non-degenerate preimages under f, then f is called an A^*-map.

2. A lemma. In [5], A. Lelek observes that a spiral about a circle is not a continuous image of a chainable continuum. In [3] L. Fearnley shows that a spiral about a triod is not, either. The following lemma generalizes both results.

Lemma 1. *If f is any continuous function such that if $\theta \geq 0$, then $f(\theta) > f(\theta + 2\pi) > 0$; and K is the set of all points in the plane with polar coordinates $(f(\theta), \theta)$, where $\theta \geq 0$; and $\bar{K}$ (the closure of K) is the continuous image of some chainable continuum; then $\bar{K} - K$ is either degenerate or an arc.*

Proof. Indeed, we show that there do not exist three numbers, θ_1, θ_2, and θ_3, such that $0 < \theta_1 < \theta_2 < \theta_3 \leq 2\pi$, and if $i \leq 3$, then the decreasing sequence

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