REPRESENTATION OF A TOPOLOGICAL GROUP ON A HILBERT MODULE

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1. In a recent paper [6] the author generalized a work of Goldstine and Horwitz [2] by considering what he called a Hilbert module over an arbitrary H^*-algebra. A Hilbert module is a right module H, over a proper H^*-algebra A, with a certain generalized scalar product defined on H with values in A.

It turns out that a Hilbert module is a very natural extension of the concept of a Hilbert space. Many important theorems in the theory of Hilbert spaces admit to generalizations in this more general system.

The present work is a continuation of [6]. We direct attention to representations of topological groups and the corresponding group algebras. In particular, we intend to generalize the following results which can be found in Chapter VI of [5].

Let G be a topological group. A positive definite function on G is a complex-valued function p defined on G such that $\sum_{i,j} \bar{\lambda}_i \lambda_j p(t_i^{-1} t_j) \geq 0$ for each finite subset $\{t_1, t_2, \cdots, t_n\}$ of G and any corresponding set $\{\lambda_1, \lambda_2, \cdots, \lambda_n\}$ of complex numbers. Theorem 1 of §30 in [5] states that each continuous positive definite function on G is of the form $p(t) = (U_t f_0, f_0)$ for some continuous unitary representation $t \rightarrow U_t$ of G on a Hilbert space H and some $f_0 \in H$. Below (Theorem 1) we give a generalization of this result. It will be seen that there are natural generalizations of both the concept of positive definite functions and the notion of unitary representation of a group.

Theorem 2 below is a generalization of Theorem 1 in §29 of [5]. It states that each *-representation of $L^1(G)$ is essentially of the form $x \rightarrow Tx = \int_G x(t) U_t dt$ for some continuous unitary representation $t \rightarrow U_t$ of G. In fact, Theorem 2 is easy to derive from its special case, stated in [5].

Theorem 3 below is a generalization of Stone's Theorem [3, 36E]. It states that each continuous unitary representation of a commutative (locally compact) group G is of the form $t \rightarrow U_t = \int_{\hat{G}} \overline{(t, \alpha)}\, dP_\alpha$ for some projection-valued measure P on the group $\hat{G}$ of characters on G.

In §5 we discuss generalizations of the Closed Graph Theorem and the Spectral Theorem for an unbounded self-adjoint operator. Both generalizations are easy consequences of their classical statements.

2. Let A be a proper H^*-algebra and let $\tau(A) = \{xy \mid x, y \in A\}$ be its trace class [7]. Then one can show (either using a technique similar to [8] or employing the fact that A is a direct sum of Schmidt classes [8]) that $\tau(A)$ is a Banach

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