LINKS AND SEIFERT FIBER SPACES

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Introduction. The purpose of this paper is to generalize the result obtained in [2] to link types. In [2] the following theorem was shown. *If the group of a knot k has a non-trivial center, k is a torus knot (including the unknotted knot).* The complementary domain of k turned out to be a Seifert fiber space. The converse is trivial. If the group of a link has a non-trivial center, then the link will be indecomposable and its complementary domain C hence irreducible [6]. Since the first homology group of a link is free, it follows from Waldhausen [8, Satz 4.1] that C is a Seifert fiber space. The converse is again trivial. We shall see, furthermore, that the fiber always generates a free cyclic group contained in the center of the link group. In nearly all cases the fibration of C can be extended to a fibration of S^3, whence from Seifert [7] a complete description of the possible links is obtained. The link groups with non-trivial center can be completely determined. Principally this paper is a simple application of the results of Seifert [7], Orlik, Vogt and Zieschang [5] and Waldhausen [8]. What may be of interest is that the discussion of the different possible link types provides a series of interesting examples, showing different possibilities in the dependence among the groups of links, their complementary domains and the link types themselves. Some groups determine the type of the complementary domains as well as the link type. For others only the complementary domain is determined, but finitely or infinitely many different link types occur. Finally, a group may determine a finite number of non-homeomorphic complementary domains, each of which allows a certain amount of link types. In all cases a complete classification of groups, complementary domains and link types is possible, thanks to the fundamental results mentioned above. (J. H. C. Whitehead [9] was the first to give examples of non-equivalent links with homeomorphic complementary domains.)

1. In [7] all Seifert fibrations of the 3-sphere S^3 are determined. Let us denote by $[\alpha, \beta]$ such a fibration, where α and β are relatively prime integers, choosing always $\alpha > 0$. α and $|\beta|$ give the order of the two (possible) exceptional fibers. $[1, 0]$ may denote the "singular" fibration of S^3 which is the trivial fibration of an unknotted solid torus in S^3, not extendable to a fibration of S^3 itself. The singular points, not included in the fibration, lie on an unknotted circle k_0 in S^3.

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