A RESULT ON UNIONS OF FLAT CELLS

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1. Introduction. In this paper we obtain the following result concerning unions of flat cells in Euclidean space.

Theorem. *Suppose* $1 \leq m < n$, $q \geq 3$, *and* $t = q, q - 1$ *or* $q - 2$. *Then there exist t-cells* $E_1, E_2, \cdots, E_n$ *in* E^q *such that*

(1) $E_1, E_2, \cdots, E_n$ *meet in a* $(q - 3)$*-cell on the boundary of each,*
(2) *any* m *of the cells* $E_1, E_2, \cdots, E_n$ *are simultaneously flat in* E^q, *and*
(3) *no* $m + 1$ *of the cells* $E_1, E_2, \cdots, E_n$ *are simultaneously flat in* E^q.

Furthermore, the flattening homeomorphism guaranteed by (2) *can be realized as the final stage of an ambient isotopy of* E^q *which is fixed outside a compact set.*

The theorem is proved in the case $q = 3$ by modifying a construction of Debrunner and Fox [3]. Multiple suspension of these examples completes the proof for the case $q > 3$. This technique (or that of coning or crossing with cubes) is a rather standard one which has been frequently used to extend 3-dimensional results.

In case $q = 3$ and $m = n - 1$, the result of the theorem is explicit in [3], while the case $n = 2$ and $t = q - 2$ has been done by Sosinskiĭ [5] and Tindell [6]. Results of Černavskiĭ [2] indicate the non-existence of such examples when the cells meet in a cell whose dimension is not $q - 3$. Related problems, in the case $n = 2$, have been studied by Cantrell [1] and Lacher [4].

By one-point compactification, the theorem is seen to be true with E^q replaced by S^q.

It is assumed that the reader is familiar with [3].

2. Definitions and notation. We regard E^q, Euclidean q-space, as the set of points $(x_1, x_2, \cdots)$ in real Hilbert space with $x_{q+1} = x_{q+2} = \cdots = 0$. We shall identify the point $(x_1, x_2, \cdots)$ in E^q with the q-tuple $(x_1, \cdots, x_q)$. Notice that $E^1 \subset E^2 \subset E^3 \subset \cdots$.

A set of points in E^q is said to be in *general position* if, whenever $1 < k \leq q + 1$, no k points of the set lie in a $(k - 2)$-hyperplane of E^q. If $r = 0, 1, \cdots$, or q, an r-simplex σ^r in E^q is the convex hull of a set $\{a^0, a^1, \cdots, a^r\}$ of $r + 1$ points in general position in E^q. A (*proper*) *face* of σ^r is a simplex determined by some (proper) subset of $\{a^0, a^1, \cdots, a^r\}$. The *interior* of σ^r, denoted Int σ^r, consists of those points in σ^r which lie in no proper face of σ^r. The *boundary* of σ^r,

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