HOLOMORPHIC FUNCTIONS OF POLYNOMIAL GROWTH ON BOUNDED DOMAINS

BY YUM-TONG SIU

In [4] R. Narasimhan proved the following theorem concerning holomorphic functions of polynomial growth: Suppose g is a holomorphic function defined on some open neighborhood of the closure of a bounded open subset Ω of $\mathbf{C}^n$. If f is a holomorphic function of polynomial growth on Ω such that $f = gh$ for some holomorphic function h on Ω, then h has polynomial growth on Ω.

It is natural to raise the following question:

(1) Suppose $(\varphi_{ij})_{1 \leq i \leq r, 1 \leq j \leq s}$ is a matrix of holomorphic functions defined on some neighborhood of the closure of a bounded open subset Ω of $\mathbf{C}^n$. Suppose $(f_i)_{1 \leq i \leq r}$ is an r-tuple of holomorphic functions on Ω having polynomial growth and for some s-tuple of holomorphic functions $(g_j)_{1 \leq j \leq s}$ we have $f_i = \sum_{j=1}^{s} \varphi_{ij}\, g_j$, $1 \leq i \leq r$, on Ω. Can we always find an s-tuple of holomorphic functions $(h_j)_{1 \leq j \leq s}$ on Ω having polynomial growth such that $f_i = \sum_{j=1}^{s} \varphi_{ij} h_j$, $1 \leq i \leq r$?

In this paper we give an affirmative answer for (1) in the case when Ω is Stein (Theorem 2 below). First we prove an infinitely differentiable analogue of our result by the partition of unity (Theorem 1 below) and then derive our result by the L^2-estimates for the $\bar{\partial}$ operator.

1. Notations. n is a fixed natural number and $m = 2n$. $\mathbf{N}$ = the set of all natural numbers. $\mathbf{N}^*$ = the set of all nonnegative integers. $\mathbf{R}_1 = \{c \in \mathbf{R} \mid c \geq 1\}$. $\mathbf{R}_+ = \{c \in \mathbf{R} \mid c > 0\}$.

$x = (x_1, \cdots, x_m)$ and $z = (z_1, \cdots, z_n)$ denote respectively points in $\mathbf{R}^m$ and $\mathbf{C}^n$. x is identified with z by $z_k = x_k + i x_{n+k}$, $1 \leq k \leq n$. $dx = dx_1 \cdots dx_m$.

$$|x| = \left(\sum_{k=1}^{m} |x_k|^2 \right)^{\frac{1}{2}}.$$

If $\alpha = (\alpha_1, \cdots, \alpha_m) \in (\mathbf{N}^*)^m$, then

$$|\alpha| = \sum_{k=1}^{m} \alpha_k, \qquad \alpha! = \prod_{k=1}^{m} (\alpha_k)!$$

and

$$D^\alpha = \partial^{|\alpha|} / \partial x_1^{\alpha_1} \cdots \partial x_m^{\alpha_m}.$$

If $E \subset \mathbf{R}^m$, then E^- = the closure of E in $\mathbf{R}^m$,

$$d(x, E) = \inf_{y \in E} |x - y|, \qquad d_E(x) = d(x, \mathbf{R}^m - E), \quad \text{and} \quad a(E) = \sup_{x, y \in E} |x - y|.$$

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