SOME APPLICATIONS OF WREATH PRODUCTS AND ULTRAPRODUCTS IN THE THEORY OF LATTICE ORDERED GROUPS

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In this paper we utilize the wreath product and ultra-product constructions to establish certain results for lattice ordered groups (henceforth, *l-groups*). We first study certain problems concerning the extension of the order on a representable l-group G to a total order. It is shown that for an abelian l-group the order can always be extended to a total order in such a way that any given set of independent or, in particular, disjoint elements are a-equivalent in the new order. Also we show that for any independent subset A of a representable l-group G, with some arbitrarily given order, the order on G can be extended to a total order which induces the given order on A. In §3 we show that any o-group can be embedded in a simple o-group. Then we take a small nibble at the problem of amalgamating l-subgroups of l-groups and succeed in amalgamating the lexicographic kernel of a lexicographic extension of an l-group with an l-isomorphic l-subgroup of an l-group for which every l-automorphism of the subgroup may be extended to a l-automorphism of the whole group.

The final section is devoted to an example that answers the question (no. 4) raised by Conrad and McAllister in [5].

I would particularly like to express my thanks to Professor Conrad for many improvements to this paper as it was originally submitted. For instance, Theorem 2.1 was originally stated for abelian groups only; also the refinements to the abelian case in Theorem 2.2 are due to Professor Conrad.

1. Terminology and definitions. The reader is referred to [1], [3] and [6] for basic facts, terminology and notation for l-groups.

Let G be an l-group. A convex l-subgroup G_γ of G is called *regular* if there is an element $g \in G$ such that G_γ is a maximal convex l-subgroup of G with respect to the property of not containing g. Then G_γ (or just γ) is called a *value* of g. We write $\Gamma(G) = \{G_\gamma : \gamma \in \Gamma\}$ for the set of regular subgroups of G. If we define an order relation on the set $R(G_\gamma)$ of the right cosets of G_γ by: $G_\gamma + a \leq G_\gamma + b$ if and only if $g + a \leq b$ for some $g \in G_\gamma$, then $R(G_\gamma)$ becomes a totally ordered set.

It is well known (cf [1], for example), that the group A of all automorphisms of a totally ordered set I is an l-group if we define $A^+ = \{f \in A : xf \geq x$ for all $x \in I\}$. Holland [Theorem 2] has shown that every l-group is l-isomorphic

Received February 19, 1968. This work was supported in part by N.R.C. Grant No. A-4044.