ON EXTENDING INTERPOLATING SETS IN THE STONE-ČECH COMPACTIFICATION

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Let X be a Hausdorff space and $C(X)$ the space of bounded continuous complex valued functions on X. If B is a subspace of $C(X)$ and F is a closed subset of X then F is an *interpolating set* for B if and only if $B \mid F = C(F)$. (Here $f \mid F$ is the restriction of $f \in C(X)$ to F and $B \mid F$ is the collection of all $f \mid F$ for f in B.) If B interpolates F, we might ask for conditions on B and/or F which imply that B interpolates a closed set K with $F \subset \text{int } K$. Bade [1] has studied this problem when $X = \beta S$, the Stone-Čech compactification of a locally compact space S, and $F = \beta S - S$. Among his many interesting results is the following.

Theorem 1. *If S is a paracompact locally compact space and A is a closed linear subspace of $C(\beta S)$ which interpolates $\beta S - S$ then A interpolates a closed neighborhood of $\beta S - S$.*

In this paper we will present an alternate proof of Bade's Theorem and we will also examine the consequences of some extra assumptions on S and A.

1. Notation and terminology. Throughout this paper S will denote a locally compact space, and A is a closed linear subspace of $C(\beta S)$ which interpolates $\beta S - S$. We denote by $C_0(S)$ those functions in $C(S)$ which vanish at infinity. Since $C(S)$ can be identified with $C(\beta S)$, $C_0(S)$ can be identified with those functions in $C(\beta S)$ which vanish on $\beta S - S$. Note that if f is any function in $C(\beta S)$ there is a g in A such that $g \mid \beta(S) \sim S = f \mid \beta(S) \sim S$. Hence $\varphi = f - g$ is in $C_0(S)$; this gives that any function in $C(\beta S)$ is the sum of a function in A and one in $C_0(S)$.

The space of complex-valued regular Borel measures on S and βS will be denoted by $M(S)$ and $M(\beta S)$ respectively. Hence, $M(\beta S) = C(\beta S)^*$ and $M(S) = C_0(S)^*$. Our standard references, a knowledge of which is assumed, will be [6], [9], and [10]. In particular we will assume the basic structure theorem for locally compact paracompact spaces [6; 241]. Because of a variety of weak topologies we will make the following distinctions. The term weak* topology is reserved for the $\sigma(M(\beta S), C(\beta S))$ topology on $M(\beta S)$. The $\sigma(M(S), C(S))$ and $\sigma(M(S), C_0(S))$ topologies on $M(S)$ will be called the C-weak* and C_0-weak* topologies respectively.

We use $A^\perp$ to denote the measures μ in $M(\beta S)$ such that $\int_{\beta S} f \, d\mu = 0$ for all f in A.

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