A STRONG HOMOTOPY EQUIVALENCE AND EXTENSIONS FOR HUREWICZ FIBRATIONS

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1. Introduction. We say that two fiber spaces $\mathfrak{F}_0$ and $\mathfrak{F}_1$ over B are *strongly fiber homotopically equivalent* if and only if there is a fiber space $\mathfrak{C}$ over $B \times I$ such that $\mathfrak{C} \mid B \times \{i\}$ is equivalent to $\mathfrak{F}_i$ for $i = 0, 1$. It is easy to see that strong fiber homotopy equivalence implies fiber homotopy equivalence. In §6 we show that these concepts coincide for compact fiber spaces. A specialization of strong fiber homotopy equivalence yields the notion of two spaces being *strongly of the same homotopy type*. We show that this coincides with the usual notion of spaces being of the same homotopy type. That is, two spaces X and Y are of the same homotopy type if and only if there is a fiber space (Z, q, I) with $q^{-1}(0) = X$ and $q^{-1}(1) = Y$.

We define an extension of a fiber space to a larger base space and investigate conditions insuring such extensions. Of particular interest are extensions to contractible spaces, e.g., the cone over the base. It is well known that any fiber space with a contractible base is fiber homotopically trivial, i.e., equivalent to a product. And, it is immediately clear that this is also true for any fiber space which can be extended to the cone over the base. We ask: Are fiber spaces which are fiber homotopically trivial characterized by the property of being extendible to the cone? In §5 we show that this extension property does characterize spaces which are strongly homotopically trivial. Thus, for compact fiber spaces we have an affirmative answer to our question.

2. Definitions and notation.

FIBER SPACES. Suppose that $\mathfrak{F} = (E, p, B)$ is a space over B [1], i.e., p is a map (continuous function of E onto B. We shall use that definition of Huerwicz fiber space (fiber space or H.f.s.) given in terms of a *lifting function* λ. We let

$$\Omega_p = \{(e, \omega) \in E \times B^I \mid p(e) = \omega(0).\}.$$

Then, $\mathfrak{F}$ is a H.f.s. if and only if there is a map $\lambda : \Omega_p \times I \to E$ such that $\lambda((e, \omega), 0) = e$ and $p\lambda((e, \omega), t) = \omega(t)$ for all (e, ω) and t. The fiber space $\mathfrak{F}$ is said to be *compact* if and only if E is compact. For $A \subset B$, $\mathfrak{F} \mid A$ will denote that fiber space $(p^{-1}(A), p_A, A)$ where $p_A = p \mid p^{-1}(A)$. If λ is a lifting function for $\mathfrak{F}$, then $\lambda_A = \lambda \mid \Omega_{p_A}$ is a lifting function $\mathfrak{F} \mid A$.

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