CONNECTED MAPPINGS

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1. Introduction. The motivation for this investigation stems primarily from the papers of Pervin and Levine [5] and Hamilton [3] concerning connected mappings and connectivity mappings, respectively. In addition, several recent articles have appeared dealing with the general area of non-continuous mappings from one topological space to another. The purpose of this paper is to pursue mainly the notion of connected mappings and their relationship to connectivity mappings. The fundamental definitions are as follows:

Definition 1.1. Let $f : X \to Y$ be a mapping of the topological space X into the topological space Y. Then f will be called *connected* provided f transforms each connected subset of X onto a connected subset of Y.

Definition 1.2. The mapping $f : X \to Y$ is a *connectivity mapping* provided the graph map $g : X \to X \times Y$, defined by $g(x) = (x, f(x))$, is connected.

It is easy to see that a connectivity mapping is always connected since the projection mapping $p : X \times Y \to Y$ is continuous and $f = p \circ g$. The converse is not true, however, as Example 3 of [3; 755] shows. This example shows that connected mappings on n-cells, $n \geq 2$, into themselves, need not leave a point fixed. As this example and results of Hildebrand and Sanderson [4] point out, there are many properties lost when going to the more general connected mappings from connectivity mappings.

Throughout, the notation for the closure of a set A is written $\mathrm{cl}(A)$ and where no confusion arises the notation $x_n \to x$ is used for a sequence of points $\{x_n\}$ converging to x.

2. General properties. As previously announced in [6], a fundamental property of connected mappings is that the inverse of a closed set under a connected mapping has closed components. This useful generalization of Theorem 2 [3; 751] for connectivity mappings is now stated for future reference. The proof is straightforward.

Theorem 2.1. *If $f : X \to Y$ is a connected mapping of the space X into the T_1 space Y and M is any closed subset of Y, then each component of $f^{-1}(M)$ is closed in X.*

Corollary 2.2. *Under the hypothesis of Theorem 2.1, if $V \subset Y$ is open, then no point of $f^{-1}(V)$ is a limit point of a finite union of components of $X - f^{-1}(V)$.*

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