SOME TOPICS IN THE THEORY OF RECURRENT MARKOV PROCESSES

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1. Summary. A recurrent Markov process $\{X_n, n \geq 0\}$ will be a Markov process with σ-finite stationary measure π, given stationary transition probability functions defined on (Ω, Σ) and satisfying one of the following successively stronger conditions:

Condition (A): $Q(x, E) = 1$ a.e. (π) on E.
Condition (B): $Q(x, E) = 1$ a.e. (π) on Ω, if $\pi(E) > 0$.
Condition (C): $Q(x, E) = 1$ for all $x \in \Omega$, if $\pi(E) > 0$,
where $Q(x, E) = P(X_n \in E$ for infinitely many $n \geq 0 \mid X_0 = x)$.

All sets discussed are measurable with respect to some σ-field clear from the context. $P(\cdot \mid \cdot)$ is conditional probability.

§§3, 4, and 5 of this article are essentially independent of each other. In §3 the investigation centers on a duality between the stochastically closed sets of a recurrent process and the invariant sets in the representation of the process on bilateral sequence space. The discussion is closely related to Theorem 2 of [12]. §4 contains a brief discussion of the concept of uniform invariance. The principal results of the paper are, perhaps, contained in §§5 and 6. Here we investigate the ratios of transition probabilities

$$\frac{\sum_{k=1}^{n} P^k(x, E)}{\sum_{k=1}^{n} P^k(y, F)}. \tag{1.1}$$

Under condition (C), it is known that the limit of (1.1) exists on the complement of a null set for each fixed E and F with $0 < \pi(F) < \infty$ [15]. In [13] it was shown that this null set cannot in general be taken fixed, independent of E and F. The idea of normality for processes is introduced here, and Theorem 7 uses this concept to get information about the convergence of (1.1) for various choices of x, y, E and F. §6 presents applications of the results of §5; we show that the limit of (1.1) exists for all x and y and all E and F, $0 < \pi(F) < \infty$ when the Markov process is obtained by taking sums of independent, identically distributed random variables, where the common distribution function belongs to a large class. Finally, we mention a few interesting open problems in §7.

General recurrent Markov processes have been dealt with, for instance, in [6], [7], [9], [10]–[13] where other references are given.

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