CONGRUENCE PROPERTIES OF THE RAYLEIGH FUNCTIONS AND POLYNOMIALS

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1. Introduction. The Rayleigh functions and polynomials have been defined in [1] and [2]; and some congruences for the polynomials have been proved there.

The object of this paper is to discuss congruence properties of the Rayleigh functions and polynomials, and to derive from them some congruences for the Bernoulli and Genocchi numbers modulo all primes in certain intervals.

The relation between the Rayleigh function $\sigma_n(\nu)$ and the polynomial $\phi_n(\nu)$ is given by (see [2; 911])

$$\phi_n(\nu) = 4^n \prod_{r=1}^{n} (\nu + r)^{[n/r]} \sigma_n(\nu). \tag{1}$$

For $\nu = \pm \frac{1}{2}$, the values of $\sigma_n(\nu)$ are (see [1; 527])

$$\sigma_n(\tfrac{1}{2}) = (-1)^{n-1} \frac{2^{2n-1}}{(2n)!} B_{2n} , \tag{2}$$

$$\sigma_n(-\tfrac{1}{2}) = (-1)^{n} \frac{2^{2n-2}}{(2n)!} G_{2n} , \tag{3}$$

where B_{2n} and G_{2n} are Bernoulli and Genocchi numbers respectively.

We remark that we have adopted here the symbol $\sigma_n(\nu)$ to represent the n-th Rayleigh function, whereas in [1] and [2] the same function was indicated by $\sigma_{2n}(\nu)$. The same holds for the n-th Rayleigh polynomial $\phi_n(\nu)$.

We introduce here a symbol for future use. If x is divisible by all primes in a closed interval $[a, b]$, then we denote this by writing

$$x \equiv 0 (\text{mod } [a, b]').$$

2. A polynomial congruence. Consider the relation (see [1; 531])

$$(\nu + 1)\sigma_n(\nu) = \sum_{k=1}^{n-1} \sigma_k(\nu + 1)\sigma_{n-k}(\nu). \tag{4}$$

If we substitute (1) in (4), then after a simplification we get

$$\phi_n(\nu) = \sum_{k=1}^{n-1} \Omega_k(n, \nu), \tag{5}$$

where

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