# ON INTERSECTIONS OF COMPACT SETS

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Every intersection of compact subsets of a Hausdorff space is again compact. For non-Hausdorff spaces this is obviously not true. Take, for example, an infinite set $X$ with two distinguished elements $a$ and $b$ and as closed sets all finite subsets of $X$ and all subsets which contain both $a$ and $b$. Then $X$ becomes a $T_1$-space such that $X - \{a\}$ and $X - \{b\}$ are compact but their intersection is not compact. During an investigation of compact subsets of arbitrary spaces with de Groot and Strecker the following questions arose:

(a) is each intersection of compact sets expressible as a finite intersection of compact sets?
(b) is every finite intersection of compact sets expressible as an intersection of two compact sets?
The following example answers both questions in the negative.

Moreover, the space $X$ which will be constructed in the following example is a compact $T_1$-space with subsets $Y, Y_1, Y_2, Y_3, \cdots$ such that

(a) $Y$ is expressible as an intersection of compact sets but not as a finite intersection of compact sets,
(b) $Y_m$ is expressible as an intersection of $m + 1$ compact sets but not as an intersection of $m$ compact sets.

*Example.* Let $n$, $m$ be natural numbers, $N = \{1, 2, \cdots\}$ the set of natural numbers, $X_m^n = \{n\} \times \{m\} \times N$, $X_m = X_m^1 \cup X_m^2 \cup \cdots \cup X_m^m$. The set

$$X = \cup\{X_m : m \varepsilon N\} \cup N \cup \{0\}$$

can be made into a compact $T_1$-space by calling a subset $A$ of $X$ closed, iff $A$ enjoys the following two properties:

$$(1) \qquad |A \cap X_m^n| \geq \aleph_0 \Rightarrow \{m, m+n\} \subset A,$$

$$(2) \qquad |\{m : A \cap (X_m \cup \{m\}) \neq \phi\}| \geq \aleph_0 \Rightarrow 0 \varepsilon A.$$

Then a subset $A$ of $X$ is compact iff $A$ enjoys (2) and

$$(3) \qquad |A \cap X_m^n| \geq \aleph_0 \Rightarrow \{m, m+n\} \cap A \neq \phi.$$

Consequently, a compact subset of $X$ which contains $X_m$ for a certain $m$ must contain $m$ or the whole set $\{m + 1, m + 2, \cdots, m + m\}$. From this it follows that $Y_m = X_m \cup X_{m+1} \cup \cdots \cup X_{m+m}$ is expressible as intersection of the

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