SOME SUBSPACES OF ANALYTIC FUNCTIONS

By Donald E. Myers

Let S be a strip in C of the form $S = \{z \mid \tau < R(z) < \eta, z \in C\}$. The main results of this paper are: an L_2 topology on square integrable analytic functions on S is stronger than uniform convergence on compact subsets of S and the imbedding of analytic functions of polynomial growth into the space of square integrable functions as a complete inductive limit space. The first result is a generalization of that in Reference [2].

In previous papers [1], [2], this author has been interested in representation theorems for distributions as analytic functions or functionals and in particular, appropriate topologies for these spaces of analytic functions. For functions analytic in the strip S, two topologies in particular are considered, uniform convergence on compact subsets of S and that generated by an L_2 norm with respect to the variable y, $z = x + iy$. The first topology can be used for arbitrary collections of analytic functions and the ring of all functions analytic in S is known to be complete in this topology. The L_2 topology is only applicable to the subspace of functions that are bounded and go to zero sufficiently fast as $|y| \to \infty$.

Notation. $S = \{z \mid \tau < R(z) < \eta\}$.

0_S denotes the algebra of all functions analytic in S with the topology of uniform convergence on compact subsets of S. Of course, 0_S is complete.

$$0_S^2 = \{f \mid f \in 0_S \text{ , } ||f|| < \infty\}$$

$$||f|| = \left[\sup \Upsilon < x < \eta \int_{-\infty}^{\infty} |f(x+iy)|^2 \, dy\right]^{\frac{1}{2}}.$$

Lemma 1. *If $\{f_n\}$ is a Cauchy sequence in 0_S^2, then it is a Cauchy sequence in 0_S.*

Proof. Suppose there exists a compact subset K of S such that $f_n - f_m$ does not converge uniformly to zero on K. Since $||f_n - f_m|| \to 0$, $|f_n(z) - f_m(z)|$ is bounded on K for each n, m. Let $g_{nm}(K) = \sup_{z \in K} |f_n(z) - f_m(z)|$. With K compact and f_n, f_m analytic, there exists for each n, m, $z_{nm} \in K$ such that $g_{nm}(K) = |f_n(z_{nm}) - f_m(z_{nm})|$. If $|f_n - f_m| \not\to 0$ uniformly on K, then for some $\epsilon > 0$, there exist two unbounded sequences $\{n_k\}$, $\{m_k\}$ such that $g_{n_k m_k}(K) > \epsilon$ for all k. Denote by K_ϵ the closure of $\{z_k\}$, $z_{m_k n_k} = z_k$. By the continuity of $|f_n(z) - f_m(z)|$, for each k there is a neighborhood $N_{z_k}(\delta_k) = N_k$ of z_k such that $|f_{n_k}(z) - f_{m_k}(z)| > \epsilon/2$ for all $z \in N_{z_k}(\delta_k)$. K is covered by the union of those neighborhoods and, since as a closed subset of a compact set, K_ϵ is compact, there is a finite subcover

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