RELATIONS AMONG SOME BASIC PROPERTIES OF NON-CONTINUOUS FUNCTIONS

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Many properties which are characteristic of continuous functions remain valid for several classes of non-continuous functions found in the literature if appropriate connectedness restrictions are imposed. These properties have been found useful in the study of such functions and often lead to results about continuous functions as well. A study of the references at the end of this paper reveals many repetitions and similarities which indicate the possibility of the existence of relationships which may unify much of this material. This motivated the investigation which led to the results reported here.

Notation. If σ is a topology for a set S, the resulting space will be denoted by (S, σ), or simply S if no confusion is likely. "A is σ–P" means the object A has property P in the space with topology σ. Unless otherwise stated, f will always denote a function on (S, σ) to (T, τ) and σ' the topology for S generated by the subbasis $\sigma \cup f^{-1}(\tau)$. $\bar{A}$ denotes the closure of the set A and, if $A \subset S$, its σ-closure will be distinguished from its σ'-closure by using $'\bar{A}$ for the latter. Recall that f is a *connected function* iff it maps σ-connected sets onto τ-connected sets, a *connectivity function* iff its graph function is connected (or equivalently [5], iff every σ-connected set is σ'-connected) and *peripherally continuous* at p iff each neighborhood of $f(p)$ contains the image of the boundary of each member of a neighborhood basis at p. Consider the following properties of a function f:

(1) $f(\bar{K}) \subset \overline{f(K)}$ if K is connected.
(1′) Components of $f^{-1}(M)$ are closed if M is.
(2) $\bar{K} = {}'\bar{K}$ if K is connected.
(3) Non-degenerate σ-connected sets have no σ'-isolated points.

Property (1) means that f "preserves" those limit points of a connected set which do not already belong to it, (2) is equivalent to saying that a σ-limit point of a connected set not containing it is also a σ'-limit point and (3) can be rephrased to read "if U and V are neighborhoods of p and $f(p)$ respectively, and K is a non-degenerate connected set containing p, then some point of $U \cap (K - p)$ maps into V." As noted below for a very irregular class of spaces (3) may not be satisfied even if $\tau = \sigma'$ (f is continuous).

Property (1) was verified and used for connected functions in separable metric spaces by Tanaka [10], and in T_2-spaces by Pervin and Levine [8]. Property (1′) and (3) were found to be valid and useful for connectivity functions in T_2 and T_1-spaces by Hamilton [4] and Stallings [9], respectively, and

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