REPRESENTATION THEOREMS FOR UNIQUELY DIVISIBLE SEMIGROUPS

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0. Introduction. A semigroup S is (uniquely) divisible if, for each $x \in S$, and each positive integer n, there exists (a unique) $y \in S$ such that $y^n = x$. In the unique case we write $y = x^{1/n}$. Uniquely divisible commutative semigroups, referred to in the sequel as UDC semigroups, have been characterized in [2]. Compact topological semigroups satisfying this hypothesis have been studied in [7] and [8]. Earlier material of a related nature occurs in [9].

In this paper, we characterize all finite dimensional compact UDC semigroups in which there exist a zero, and identity, and no other idempotents. If $H(1) = 1$, then such a semigroup is the one-point compactification of a closed positive cone in E^n; the characterization in the more general case is in terms of the one above and $H(1)$. An extremely helpful fact in these cases is the existence of (non-zero) cancellation in S. This follows directly from the characterization in [2]; it may also be deduced directly, using the compactness and a result from [9], although this extra hypothesis is not necessary.

§3 treats finite dimensional compact UDC semigroups in which the set of idempotents is totally disconnected. In particular, it is shown that there exist sufficiently many continuous homomorphisms into the complex disk to separate points. See [5] for related results.

We shall have occasion to use the following concept due to C. E. Clark [3].

Definition 1. Let $\{I_1, \cdots, I_n\}$ be a family of subsemigroups of a commutative semigroup S, each iseomorphic to [0, 1] under real number multiplication, and having a common identity and a common zero. The collection $\{I_1, \cdots, I_n\}$ is *algebraically independent* if there does *not* exist a partition of the integers $\{1, \cdots, n\}$ into disjoint, non-empty subsets A and B such that

$$\prod_{j \in A} I_j \cap \prod_{j \in B} I_j$$

contains a non-idempotent element.

It has been shown in [7] that, if each group in S is trivial, and if $x \notin E$, then $[x] = \{x^r : r \text{ is a positive rational number}\}^*$ is iseomorphic to [0, 1] under real number multiplication. We shall refer to any such semigroup as a *usual unit interval*. Hence, if t is any non-negative real number, then x^t is a well-defined element of $[x]$. It follows readily that the standard rules of exponents are satisfied, and we shall use these without further comment.

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