THE COEFFICIENTS IN AN ASYMPTOTIC EXPANSION AND CERTAIN RELATED NUMBERS

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1. Introduction. Put

$$e^{nz} = \sum_{r=0}^{n} \frac{(nz)^r}{n!} + \frac{(nz)^n}{n!} S_n(z),$$

where n is a positive integer and z is an arbitrary complex number. Buckholtz [1] proved that, for $k \geq 1$,

$$S_n(z) = \sum_{r=0}^{k-1} n^{-r} U_r(z) + O(n^{-k})$$

uniformly in a certain region of the z-plane. The coefficients $U_r(z)$ are determined by means of

$$(1.1) \qquad U_r(z) = (-1)^r \left(\frac{z}{1-z} \frac{d}{dz}\right)^r \frac{z}{1-z}.$$

It follows from (1.1) that

$$(1.2) \qquad U_r(z) = (-1)^r \frac{Q_r(z)}{(1-z)^{2r+1}},$$

where, for $r \geq 1$, $Q_r(z)$ is a polynomial of degree r with positive integral coefficients.

The writer [2] showed that

$$(1.3) \qquad Q_k(z) = (1-z)^{2k+1} \sum_{n=1}^{\infty} z^n S(n+k, n),$$

where $S(n+k, n)$ is a Stirling number of the second kind:

$$S(n+k, n) = \frac{1}{n!} \sum_{j=0}^{n} (-1)^{n-j} \binom{n}{j} j^{n+k}.$$

If we put

$$Q_k(z) = \sum_{n=1}^{k} a_{kn} z^n \qquad (k \geq 1),$$

then the a_{kn} satisfy the recurrence

$$(1.4) \qquad a_{kn} = n a_{k-1,n} + (2k-n) a_{k-1,n-1} \qquad (1 < n \leq k).$$

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