FOURIER SERIES OF DIFFERENTIABLE FUNCTIONS ON SU(2)

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Let $G = SU(2)$ and for each integer $n > 0$ let χ_n be the n-dimensional irreducible character of G, (see [3] page 151: our χ_n is Weyl's χ_{n-1}). Any function f in $L^1(G)$ has a Fourier series

$$(1) \qquad f \sim \sum_{n=1}^{\infty} P_n f \qquad P_n = f * n\chi_n$$

where $*$ denotes convolution. Let $C^1(G)$ be the space of continuously differentiable functions on G. Let $b \in G$. We will say that a function f on G is of class C^1 at b if f agrees with a function in $C^1(G)$ on some neighborhood of b. The main results of this paper are contained in the following two theorems.

THEOREM A. *If $f \in C^1(G)$, then the Fourier series for f converges to f uniformly. There exists a function $g \in C^1(G)$ whose Fourier series does not converge absolutely.*

THEOREM B. *If $f \in L^2(G)$ and f is of class C^1 almost everywhere, then the Fourier series for f converges almost everywhere to f. There is a function $g \in L^1(G)$ such that g is of class C^1 almost everywhere, but the Fourier series for g diverges everywhere.*

If f is as in Theorem B, I know of no relation between the set of points where f is not of class C^1 and the set of points where the Fourier series for f diverges. The two sets can be non-void and disjoint (see [2 Lemma 3.30]).

If $f \in L^1(G)$, we define the *Riemann Lebesgue set* $r(f)$ of f to be $\{a \in G: \lim_{n\to\infty} P_n f(a) = 0\}$. The proof of Theorem B uses the following theorem, which is a generalization of the Riemann Localization Theorem.

THEOREM C. *Let $f \in L^1(G)$ be a function which vanishes on a neighborhood of $b \in G$. Then the Fourier series $\sum_{n=1}^{\infty} P_n f(b)$ converges to 0 if and only if b is in the Riemann Lebesgue set of f.*

Proof of Theorem A. Let $\mathfrak{g}$ be the Lie algebra of left invariant vector fields on G. For any $f \in L^1(G)$ and any $N > 0$ let $S_N f = \sum_{k=1}^{N} P_k f$ be the N-th partial sum of the Fourier series for f.

LEMMA 1. *Let D_1, D_2, D_3, be a basis for $\mathfrak{g}$ which is orthonormal with respect to the Killing form on $\mathfrak{g}$. Then there exists a constant K such that*

$$\|S_N f\|_\infty \le \|f\|_\infty + K(\|D_1 f\|_\infty + \|D_2 f\|_\infty + \|D_3 f\|_\infty)$$

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