STEPANOFF AND WEYL AP FUNCTIONS ON LOCALLY COMPACT GROUPS

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1. Introduction. We consider here a theory of almost periodic functions which generalizes the classical theories of Stepanoff, [10], and H. Weyl, [12], to locally compact T_0-topological groups (= LC groups). As pointed out in [2], both of these classical theories may be considered as dealing with functions which lie in the closure of the continuous trigonometric polynomials, where this closure is formed with respect to an appropriate norm. The Stepanoff and Weyl norms are given, respectively, by

$$||f||_L^{S^p} = \sup_x \left[\frac{1}{L}\int_x^{x+L} |f(t)|^p\, dt\right]^{1/p},$$

and

$$||f||^{W^p} = \lim_{L\to\infty} ||f||_L^{S^p},$$

where $L > 0$, $p \geq 1$. The resulting classes of functions, $\{S_L^p - AP\}$ and $\{W^p - AP\}$, may be described in structural terms which reveal their "almost periodic" character.

Let G be an LC group and let $\tau(G)$ denote the set of continuous "trigonometric polynomials" on G, that is, the set of linear combinations of entries in the continuous, irreducible, unitary, finite-dimensional representations of G. Our generalized Stepanoff functions $\{S_U^p - AP\}$ are obtained by closing $\tau(G)$ with respect to the norm

$$||f||_U^{S(p)} = \sup_{x,y\in G} \left[\frac{1}{\mu(Uy)}\int_{xUy} |f|^p\, d\mu\right]^{1/p},$$

where U is a bounded open subset of G and μ is left Haar measure. As in the classical case, $\{S_U^p - AP\}$ is independent of U and we are able to characterize its members in terms of the "relative density" of their "ϵ-periods" and in terms of the total boundedness of their translates.

Our Weyl norm $||\ ||_\Phi^{W(p)}$ is defined in a more complicated fashion and the resulting $W_\Phi^p - AP$ functions are characterized in terms of their internal structure. The family $\{W_\Phi^p - AP\}$ depends on the real parameter $p \geq 1$ and on the "Bohr net" Φ in a fashion which will be described. If G is the additive group of real numbers with the usual topology and Φ is chosen appropriately,

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