SOLVING INTEGRAL EQUATIONS BY ITERATION

By J. V. Herod

H. S. Wall [7, Theorem 1] found that, if n is a positive integer, there is a reversible function $\mathcal{E}_0$ from the class Φ_n of all n-by-n matrices F of complex functions defined on the real line S, continuous and of bounded variation on each interval, such that $F(0) = 0$ to a class H_n which is established by the condition that

$$W(x, z) = 1 + \int_x^z dF \cdot W(I, z) \quad \text{for each} \quad \{x, z\} \quad \text{in} \quad S \times S. \tag{1}$$

The W is obtained from the F by successive substitution in (1) generating a convergent Peano-series expansion.

Using the Cauchy left and right integrals, J. S. Mac Nerney [2, Theorem 4.3] extended this idea to obtain a reversible function $\mathcal{E}$ from a class $\mathcal{OA}$ onto a class $\mathcal{OM}$ such that if V is in $\mathcal{OA}$ and $W = \mathcal{E}(V)$ then

$$W(x, z) = 1 + (L) \int_x^z W(x, I) \cdot V \tag{2}$$

and

$$W(x, z) = 1 + (R) \int_x^z V \cdot W(I, z) \quad \text{for each} \quad \{x, z\} \quad \text{in} \quad S \times S. \tag{3}$$

Mac Nerney [2, Theorem 6.1 and 6.2] showed that this W might also be obtained from the V by iteration in (2) or (3).

In [3, §§5, 6, and 7] Professor Mac Nerney postulated four axioms for an ordered pair $\{K_1, K_2\}$ whereby the systems

$$W(x, z) = 1 + K_1[W(x, I)](x, z) \quad \text{and} \quad W(x, z) = 1 + K_2[W(I, z)](x, z)$$

might be reduced to this left-right context. The present author establishes in this paper that there is a convergence theorem for the series generated by iteration in K_1 or K_2. It will be shown that, in some cases, the series obtained from K_1 is the same as the series obtained from K_2. Indeed, this will be seen to be the case when K_1 and K_2 are defined in terms of the left and right integrals respectively or the interior [6; 123] and Young [8] integrals respectively (see [3, §8]). The techniques that are developed will be applied to a nonlinear integral operator to extend the results of [5].

Let us suppose that S is the set of real numbers and that R is a ring with unity on which there is defined a real-valued norm with respect to which R is

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