TWISTED MATRIX UNITS SEMIGROUP ALGEBRAS

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By a matrix units semigroup (of degree n) is meant a subsemigroup S of the usual semigroup of $n \times n$ matrix units. To insure the existence of an identity in the semigroup algebra of S we shall always assume that S contains all idempotent matrix units $e_{11}, e_{22}, \cdots, e_n$.

The primary objective of this paper is to prove that a finite-dimensional algebra A with identity over an algebraically closed field is isomorphic to a twisted matrix units semigroup algebra if and only if

(i) A has a finite ideal lattice; and

(ii) the left annihilator of every ideal of A is generated (as a left ideal) by an idempotent.

A ring which has an identity and satisfies (ii) we call *quasi-Baer*. (Recall that Kaplansky [4] defined a *Baer ring* to be a ring with identity in which the left annihilator of every *subset* is generated by an idempotent.) In §1 we characterize quasi-Baer, Artinian rings as those Artinian rings which satisfy the following property: If e is a primitive idempotent of A and if $x \varepsilon A$ such that $xeAex = 0$, then $xe = 0$ or $ex = 0$. We show that those ideals of such a ring A which are left annihilators of left ideals form a finite distributive sublattice of the lattice of all ideals of A, and that every finite distributive lattice can be represented in this fashion.

In §3 we prove that an Artinian ring A has zero right (left) singular ideal if and only if the left (right) annihilator of the radical of A is generated as a left (right) ideal by an idempotent. In particular, quasi-Baer, Artinian rings have zero right and left singular ideals.

1. Quasi-Baer, Artinian Rings. If S is a subset of a ring A, we let $l(S)$ [$r(S)$] denote the left [right] annihilator of S in A.

LEMMA 1. *If A is a ring with identity, then the following statements are equivalent:*

(a) *A is quasi-Baer.*

(b) *The left annihilator of every left ideal of A is generated by an idempotent.*

(c) *The right annihilator of every right ideal of A is generated by an idempotent.*

(d) *The right annihilator of every ideal of A is generated by an idempotent.*

Proof. For any left ideal L of A, LA is an ideal of A and $l(L) = l(LA)$ since A has an identity. Hence it is clear that $(a) \Leftrightarrow (b)$. Similarly, $(c) \Leftrightarrow (d)$. By symmetry it suffices to prove that $(b) \Rightarrow (c)$: If R is a right ideal then $r(R)$

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