ON THE GROUP OF CONFORMAL TRANSFORMATIONS OF A COMPACT RIEMANNIAN MANIFOLD. II

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1. Introduction. Let M^n be a connected Riemannian manifold of dimension n, and $C_0(M^n)$, $I_0(M^n)$ the largest connected groups of conformal transformations and isometries of M^n respectively. In a previous paper [2], the author established

Theorem 1. *Let R_{hijk}, $R_{ij}(h, i, j, k = 1, \cdots, n)$ be respectively the Riemann and Ricci tensors of a compact Riemannian manifold $M^n(n > 2)$ with positive constant scalar curvature R, and suppose that*

$$P^pQ^q = C = const. , \tag{1}$$

$$C\left[\frac{2p}{P} + \frac{(n-1)q}{Q}\right] = \frac{2^p(p+q)R^{2(p+q-1)}}{n^{p+q-1}(n-1)^{p-1}}, \tag{2}$$

where p, q are nonnegative integers and not both zero, and

$$P = R^{hijk}R_{hijk} , \qquad Q = R^{ij}R_{ij} . \tag{3}$$

If $C_0(M^n) \neq I_0(M^n)$, then M^n is isometric to a sphere.

It should be noted that when $p = 0$, $q = 1$, or $p = 1$, $q = 0$, Equation (2) is an identity, and for the first special case Theorem 1 is due to Lichnerowicz [3]. Furthermore, we still have the open question: When $p = q = 0$, is Theorem 1 still true?

On the other hand, Obata [4] obtained

Theorem 2. *Let $M^n(n \geq 2)$ be a complete Riemannian manifold with metric tensor g_{ij}, and ∇ the operator of covariant derivation of M^n. If M^n admits a nonconstant function ρ such that $\nabla_i\nabla_j\rho = -c^2\rho g_{ij}$, where c is a positive constant, then M^n is isometric to an n-sphere of radius $1/c$.*

Very recently, by making use of Theorem 2, Yano [5] proved

Theorem 3. *Suppose that a compact orientable Riemannian manifold $M^n(n > 2)$ with constant R admits an infinitesimal nonhomothetic conformal transformation v so that*

$$L_vg_{ij} = 2\phi g_{ij} , \qquad \phi \neq const. , \tag{4}$$

where L_v is the operator of the infinitesimal transformation v. If

$$\int_{M^n} T_{ij}\phi^i\phi^j \, dA_n \geq 0,$$

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