SOME PROPERTIES OF RELATIVE INVARIANTS ON BOUNDED DOMAINS

By Kyong T. Hahn

1. Introduction. Let D be any bounded domain in the space $\mathbf{C}^n$ of n complex variables $z = (z^1, z^2, \cdots, z^n)$. The Bergman kernel function $K_D(z, \bar{t})$ of D is a holomorphic function of $(z, \bar{t})$ in the product domain $D \times D$ and belongs to the space $L^2(D)$ of square integrable holomorphic functions of D. Moreover,

$$K_D(z, \bar{t}) = (K_D(t, \bar{z}))^{-}, \qquad K_D(z, \bar{z}) > 0 \tag{1}$$

and the reproducing property

$$f(z) = \int_D K_D(z, \bar{t}) f(t)\, d\omega_t \tag{2}$$

holds for any $f(z)$ in $L^2(D)$, where $d\omega_t$ denotes the euclidean volume element at the point $t \in D$. It is well known that the kernel function defines the Bergman metric

$$ds_D^2(z) = T_{\alpha\bar{\beta}}\, dz^\alpha\, d\bar{z}^\beta, \tag{3}$$

which is invariant under biholomorphic mappings of D; here the summation convention is used, and the metric tensor is given by

$$T_{\alpha\bar{\beta}} = \partial^2 \log K_D / \partial z^\alpha\, \partial \bar{z}^\beta, \qquad K_D = K_D(z, \bar{z}).$$

In connection with the Bergman metric various domain functions may be defined which enable us to study geometry on bounded domains. A real-valued domain function $Q_D(z, \bar{z})$ is a *relative invariant* of D if

$$Q_D(z, \bar{z}) = Q_B(w, \bar{w})\, |J_f(z)|^2 \quad \text{on} \quad D, \tag{4}$$

and an *invariant* of D if

$$Q_D(z, \bar{z}) = Q_B(w, \bar{w}) \quad \text{on} \quad D \tag{5}$$

under any biholomorphic mapping $w = f(z)$ of D onto B, where $J_f(z)$ denotes the Jacobian of the mapping. Well-known relative invariants are $K_D(z, \bar{z})$, $T_D(z, \bar{z}) \equiv \det(T_{\alpha\bar{\beta}})$ and $R_D(z, \bar{z}) \equiv \det(R_{\alpha\bar{\beta}})$, where $R_{\alpha\bar{\beta}}$ are the components of the Ricci curvature tensor of the metric (3), given by

$$R_{\alpha\bar{\beta}} = -\partial^2 \log T_D / \partial z^\alpha\, \partial \bar{z}^\beta, \qquad T_D = T_D(z, \bar{z})([7]). \tag{6}$$

The quotient

$$I_D(z, \bar{z}) = K_D(z, \bar{z}) / T_D(z, \bar{z}) \tag{7}$$

defines an invariant of D.

Received June 22, 1966.