ACTION OF T^n ON COHOMOLOGY LENS SPACES

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1. Introduction. D. Montgomery and G. D. Mostow [6] have shown that if the toroid T^n acts effectively on a $(2n + 1)$-spherelike cohomology manifold X over Z, then T^n has exactly 2^n isotropy subgroups, e, T^1, $\cdots$, T^n and their direct products, where e is the identity element. That is, there is no isotropy subgroup with finite order except e. It is shown here that if T^n acts effectively on a $(2n + 1)$-lens-like cohomology manifold Y over Z, then T^n has an isotropy subgroup with finite order besides e. Also, it is shown that if T^n acts effectively on a cohomology lens $(2n + 1)$-space Y over Z_p, then the fixed point set $F(T^n, Y)$ is a cohomology lens $(2r + 1)$-space over Z_p, where $r \leq n$.

See [1], [9] and [10] for notations used here.

Let S^{2n+1} be a $(2n + 1)$-dimensional unit sphere in Euclidean $(2n + 2)$-space defined in terms of $(n + 1)$ complex coordinates $(Z_0, \cdots, Z_n)$ satisfying $Z_0\bar{Z}_0 + \cdots + Z_n\bar{Z}_n = 1$.

Let $p \geq 2$ be a fixed integer, and $q_1, \cdots, q_n$ be n integers relatively prime to p. We define an action α on S^{2n+1} onto itself by $\alpha(t, (Z_0, \cdots, Z_n)) = (e^{2\pi i/p}Z_0, e^{2\pi i q_1/p}Z_1, \cdots, e^{2\pi i q_n/p}Z_n)$. Then t generates a fixed point free cyclic group $Z_p(t)$ of rotations of S^{2n+1} of order p. The orbit space $S^{2n+1}/Z_p(t) = L_{2n+1}(p; q_1, \cdots, q_n)$ is an orientable $(2n + 1)$-dimensional manifold called a *lens space*. If $\pi : S^{2n+1} \to S^{2n+1}/Z_p(t)$ is the projection map and $g \in Z_p(t)$, then $\pi g(x) = \pi(x)$ for $x \in S^{2n+1}$. $Z_p(t)$ is the *group of deck* (or *covering*) transformations since π is a covering map.

We shall denote by $L_{2n+1}(p)$ the lens space $L_{2n+1}(p; 1, \cdots, 1)$, and by a *cohomology lens* $(2n + 1)$-*space* Y we mean a locally compact Hausdorff $(2n + 1)$-space whose cohomology ring over Z or Z_p is the same as that of $L_{2n+1}(p)$ over Z or Z_p, where Z is the ring of integers and $Z_p = Z/pZ$, p an odd prime number.

The ring structure of a cohomology lens space over Z_p with respect to the coefficients Z_p is $H^*(L_{2n+1}(p); Z_p) = \Lambda[a] \otimes Z_p[x]/(x^{n+1})$, where $\Lambda[a]$ is an exterior algebra on one generator a of degree 1, $Z_p[x]/(x^{n+1})$ is a polynomial algebra on one generator of degree 2 and truncated in dimension $2n + 2$ (see [9] for more details).

The ring structure with respect to the coefficients Z is a polynomial algebra on one generator of degree 2 and truncated in dimension $2n + 2$. The generator in dimension $2n + 1$ can be chosen to be the fundamental cocycle of the manifold.

For the combinatorial equivalence and homotopy type classifications of lens spaces see [7].

1. Let Y be a $(2n + 1)$-dimensional compact, arcwise connected, arcwise locally connected, and simi-locally 1-connected space whose $\pi_1(Y) = Z_p$ and

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