ON THE MEAN VALUE OF HAAR MEASURABLE ALMOST PERIODIC FUNCTIONS

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1. Introduction. Harald Bohr showed [1; 45] that the mean value of a continuous complex-valued almost periodic function on the real line is given by

$$Mf = \lim_{T\to\infty} \frac{1}{2T} \int_{-T+a}^{T+a} f(x)\, dx,$$

uniformly in real numbers a. We are concerned with generalizing this formula to Haar measurable complex-valued von Neumann almost periodic functions defined on a locally compact topological group. Such functions are necessarily continuous, in fact, (left and right) uniformly continuous.

By an LC group we mean a locally compact T_0-topological group. If G is an LC group, let $\alpha(G)$ denote the continuous almost periodic functions on G and μ be a left Haar measure on the Borel sets of G (i.e., μ is defined on the σ-algebra generated by the closed sets of G). In 1943 Kawada [6] showed that if G is a connected abelian LC group, then there is a sequence $\{U_n\}_{n=1}^{\infty}$ of subsets of G such that

(1) each U_n is bounded (i.e., $\bar{U}_n$ is compact) and open,

(2) $U_1 \subset U_2 \subset \cdots$,

(3) $\bigcup_{n=1}^{\infty} U_n = G$,

(4) $\lim_{n\to\infty} \frac{1}{\mu(U_n)} \int_{U_n} f\, d\mu = Mf \quad \text{for all} \quad f \in \alpha(G).$

He remarked that the same result holds for every connected LC group G such that $\alpha(G)$ separates points by virtue of the Freudenthal–Weil structure theorem for such groups [2]; [9; 126–129]. In 1948 Lyubarskiĭ [7] proved the same result as Kawada by a more direct method. In 1963 Hewitt and Ross [5, 18.11–18.14] showed that if G is any σ-compact abelian LC group, then there is a sequence $\{U_n\}_{n=1}^{\infty}$ of subsets of G satisfying (1), $\cdots$, (4). The methods of proof used above depend on the fact that there exists a sequence $\{A_n\}_{n=1}^{\infty}$ of measurable subsets of positive finite measure in G such that

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