# SPAN OF LINEAR COMBINATIONS OF DERIVATIVES OF POLYNOMIALS

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**1.** Recently Robinson [2] has considered the problem of determining the maximum span $\sigma_k(P)$ (the distance between the largest and smallest zero) of the $k$-th derivative of polynomials $P(x)$ of degree $n$ having $n$ real zeros $x_1$, $x_2, \cdots, x_n$ satisfying

$$|x_i| \leq 1, \qquad 1 \leq i \leq n. \tag{1.1}$$

Later we obtained [1] analogous results for the maximum span when the zeros of the polynomials were subjected to either of the following conditions:

$$x_i \geq 0 \quad (1 \leq i \leq n) \qquad \text{and} \qquad \sum_{i=1}^{n} x_i \leq n, \tag{1.2}$$

$$\sum_{i=1}^{n} x_i^2 \leq n. \tag{1.3}$$

These conditions are suggested by similar investigations connected with classical orthogonal polynomials [3; 140 and 379]. Here we consider differential operators of the form

$$h(D) = \prod_{j=1}^{m} (D + \alpha_j), \qquad \left(D \equiv \frac{d}{dx}\, ; \quad \alpha_j \text{ constants}\right), \tag{1.4}$$

and propose the problem of determining the maximum span (or the polynomial yielding the maximum span) of $h(D)P(x)$, when the zeros of $P(x)$ satisfy (1.1) or (1.2). The case when all the $\alpha_j$'s are zero has been treated in [1] and [2]. When all the $\alpha_j$'s are of the same sign and the zeros of $P(x)$ satisfy (1.1), we are able to apply the technique developed by Robinson in [2] to show that the polynomials yielding the maximum span of $h(D)P$ are of the form $c(x - 1)^p(x + 1)^q$, $p + q = n$, $c$ constant. We determine $p$ and $q$ uniquely for the special case $h(D) = D + \alpha$. When all the $\alpha_j$'s are negative and the zeros of $P(x)$ satisfy (1.2), we obtain the complete solution of the problem if $m \leq n - 2$. We solve the second problem for $h(D) = D + \alpha$ also if $\alpha > 0$.

**2. A lemma.** Let $h(t)$ be a given polynomial of exact degree $m$ with all its zeros real and positive, i.e.,

$$h(t) = \prod_{j=1}^{m} (t + \alpha_j), \qquad \alpha_j < 0, \qquad 1 \leq j \leq m. \tag{2.0}$$

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