INFINITE INTERVAL BOUNDARY VALUE PROBLEMS FOR $y'' = f(x,y)$

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1. Introduction. In this paper we shall be concerned with the existence and uniqueness of solutions of boundary value problems of the following three types:

$$\text{(I)} \qquad \begin{cases} y'' = f(x, y) \\ y(0) = -\alpha, \qquad \alpha > 0 \\ y'(x) \geq 0, \qquad y(x) \leq 0 \quad \text{on} \quad [0, +\infty), \end{cases}$$

$$\text{(II)} \qquad \begin{cases} y'' = f(x, y) \\ y(0) = \alpha, \qquad \alpha \text{ real} \\ y(x) \quad \text{bounded on} \quad [0, +\infty), \end{cases}$$

and

$$\text{(III)} \qquad \begin{cases} y'' = f(x, y) \\ y(x) \quad \text{bounded on} \quad (-\infty, +\infty). \end{cases}$$

Subsets of the following conditions on $f(x, y)$ will be imposed as needed:

(1) $f(x, y)$ is continuous on $S = \{(x, y) \mid x \in I_1, y \in I_2\}$, with I_1 and I_2 intervals associated with the boundary value problem being considered.
(2) $f(x, y)$ is nondecreasing in y for each fixed $x \in I_1$.
(3) $f(x, y)$ is strictly increasing in y for each fixed $x \in I_1$.
(4) $f(x, 0) \equiv 0$ on I_1.
(5) $|f(x, 0)| \leq M$ on I_1.
(6) $|f(x, y) - f(x, 0)| \geq \beta\, |y|$, $\beta > 0$, for $(x, y) \in S$.
(7) there exists an η, $0 < \eta < 1$, and a function $\delta(p)$ defined on $(\eta, 1)$ such that
  (i) $(1 - p)^{[1/(1-p)]} \leq \delta(p) \leq 1$ for all $p \in (\eta, 1)$
  (ii) $\eta < p < q < 1$ implies $\delta(q) < \delta(p)$
  (iii) $\lim_{p \to 1} \delta(p) = 0$
  (iv) $-|y|^p \leq f(x, y)$ for $x \geq 0$, $-\delta(p) \leq y \leq 0$.

We shall prove the following results concerning the boundary value problem (I):

THEOREM 3.1. *If $f(x, y)$ satisfies conditions* (1), (2), *and* (4) *on $S_1 = \{(x, y)$: $x \geq 0, y \leq 0\}$, then* (I) *has a unique solution.*

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