ON THE EQUIVALENCE OF SMALL AND LARGE INDUCTIVE DIMENSION IN CERTAIN METRIC SPACES

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1. Introduction. Until fairly recently, one of the major unsolved problems in point set topology was that of determining whether the concepts of small and large inductive dimension are equivalent in metric spaces. P. Roy [6] settled this in the negative in 1962 by constructing a complete metric space X with ind $X = 0$ and Ind $X = 1$. The problem remains of determining for which metric spaces the two do coincide. Morita [4] found a partial solution by showing that ind $X =$ Ind X for metric spaces with the star-finite property. The second author, in his doctoral dissertation [3], defined the concept of total paracompactness and showed that ind $X =$ Ind X for totally paracompact metric spaces. He also exhibited an example, namely, the cone over an uncountable discrete space, of a totally paracompact metric space without the star-finite property and raised the question as to whether total paracompactness is a generalization of the star-finite property. The answer is in the negative as follows from an announcement of Corson, McMinn, Michael, Nagata [2]. The irrationals form a metric space with the star-finite property that is not totally paracompact.

The purpose of this paper is to give a topological property which, in arbitrary spaces, generalizes the notion of total paracompactness and the star-finite property and which, in metric spaces, guarantees equality of large and small inductive dimension.

2. Definitions and notations. If G is a collection of point sets, then G^* denotes the union of all the elements of G.

The topological space X is *totally paracompact* if and only if every basis for X has a locally finite subcollection covering X. The statement that X is *order totally paracompact* means that if G is a basis for X, then there is an ordered collection $(H, <)$ of open sets covering X such that

(i) if $h \in H$, there is an element g of G such that $h \subset g$ and $B(h) \subset B(g)$, ($B(M)$ denotes the boundary of M)

(ii) if $h \in H$, then the collection of all elements of H preceding h is locally finite at each point of $\bar{h}$.

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